Dynamic visual proofs in mathematics education

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Visual proofs

- History

- Present

- Static visual proofs

- Dynamic visual proofs

Research

- Design

- First insights

- Future plans

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- Pictures and diagrams always play an important role in the process of understanding various mathematics features.

- From the remote past pictures serve as visual proofs – "proofs without words"

Earthen plate, Babylonia, about 1700 BC

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Greek coin   (Aeginetic drachm) 400 BC

Euclid's Elements
Book II
Proposition 4

"If a straight line is cut at random, the square on the whole equals the squares on the segments plus twice the rectangle contained by the segments."

Euclid's Elements
Book II
Proposition 4
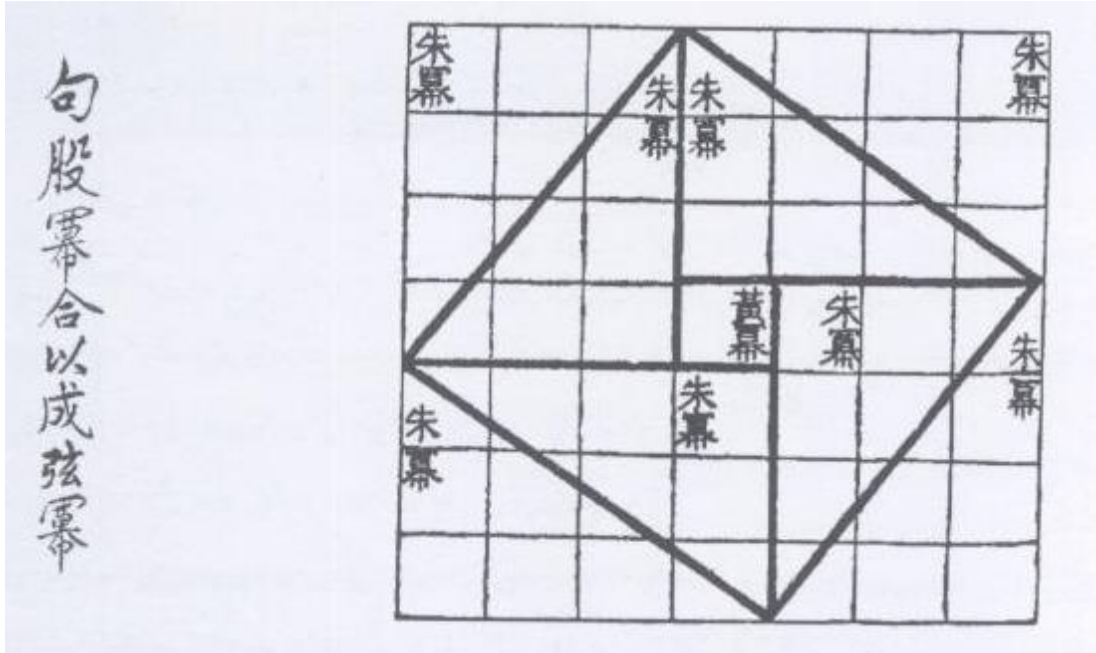
Greek coin   (Aeginetic drachm) 400 BC

$$(A + B)^2 = A^2 + 2AB + B^2$$

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Very frequent were the visual proofs of the Pythagorean theorem

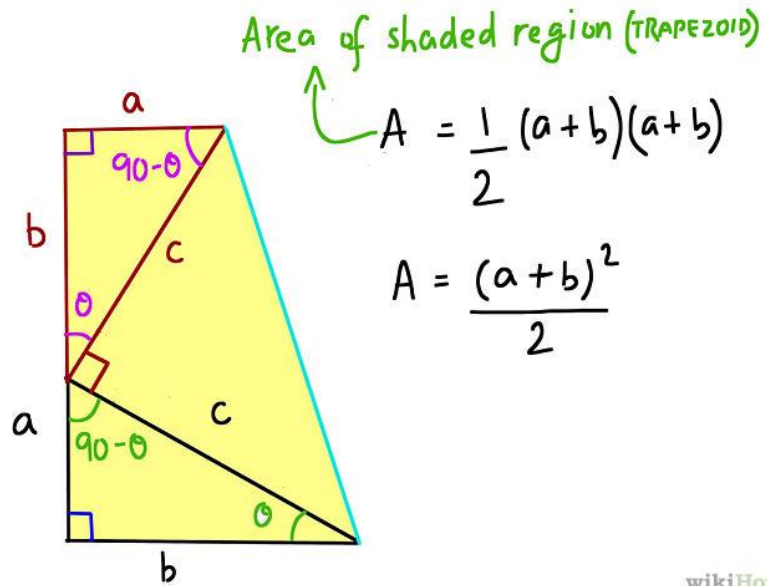The oldest known proof of Pythagoras' theorem, China, 600 BC
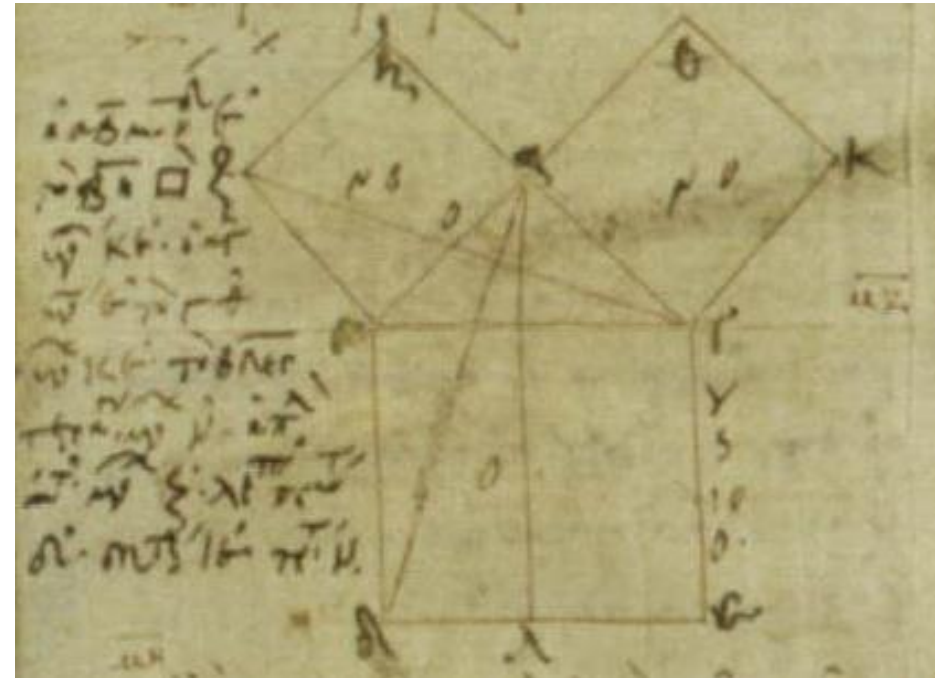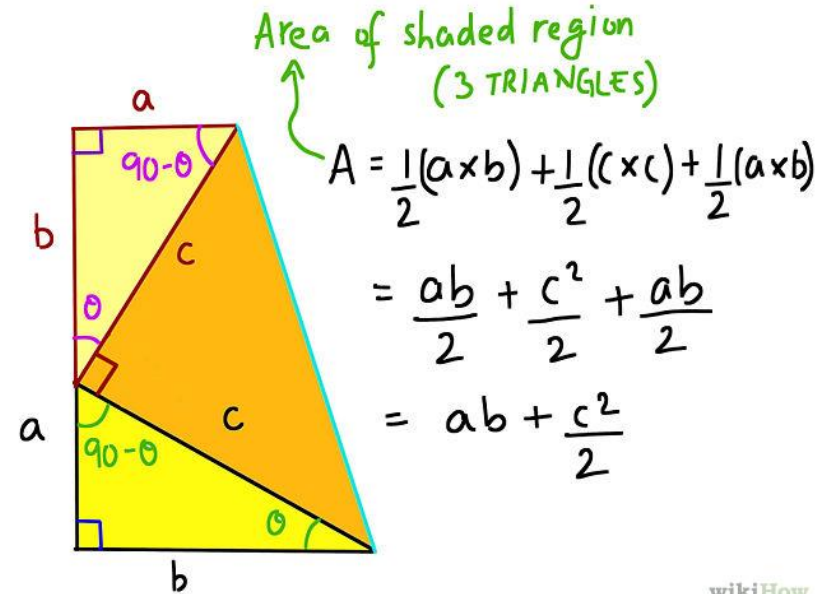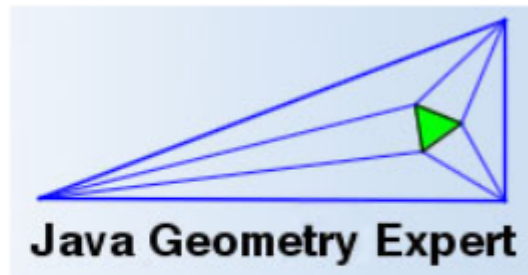

Euclid's Elements, Greek edition, 9th century,

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In a certain period the search for visual proof of the Pythagorean theorem has become a popular parlour game in mathematical communities

Area of shaded region (TRAPEZOID)

$$A = \frac{1}{2}(a+b)(a+b)$$

$$A = \frac{(a+b)^2}{2}$$

Area of shaded region (3 TRIANGLES)

$$A = \frac{1}{2}(a \times b) + \frac{1}{2}(c \times c) + \frac{1}{2}(a \times b)$$

$$= \frac{ab}{2} + \frac{c^2}{2} + \frac{ab}{2}$$

$$= ab + \frac{c^2}{2}$$

Garfield (20th President of the USA), 1876
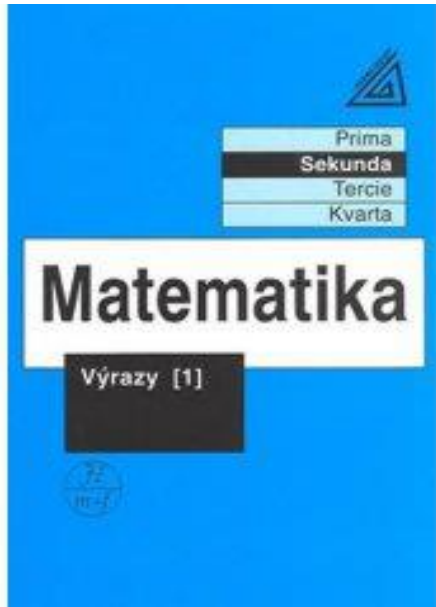(http://www.wikihow.com/Do-Garfield's-Proof-of-the-Pythagorean-Theorem)

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- Roger B. Nelsen: Proofs without Words:

  Exercises in Visual Thinking, 1993

- Roger B. Nelsen: Proofs Without Words II:
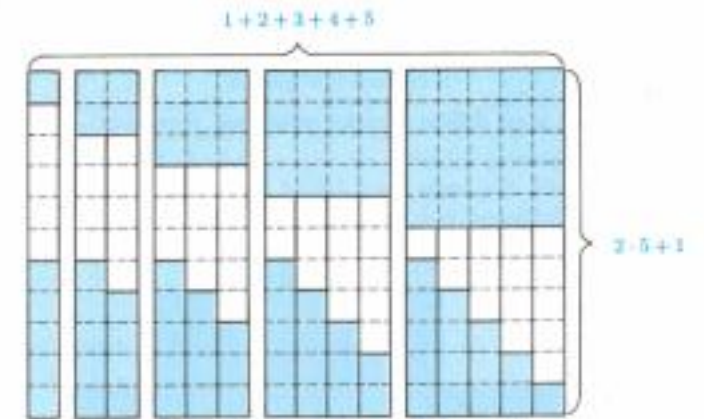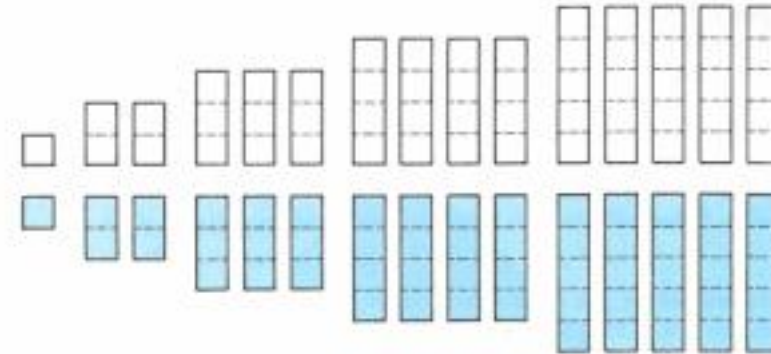
  More Exercises in Visual Thinking, 2001

http://www.cut-the-knot.org/

http://www.cs.wichita.edu/~ye/

http://demonstrations.wolfram.com/

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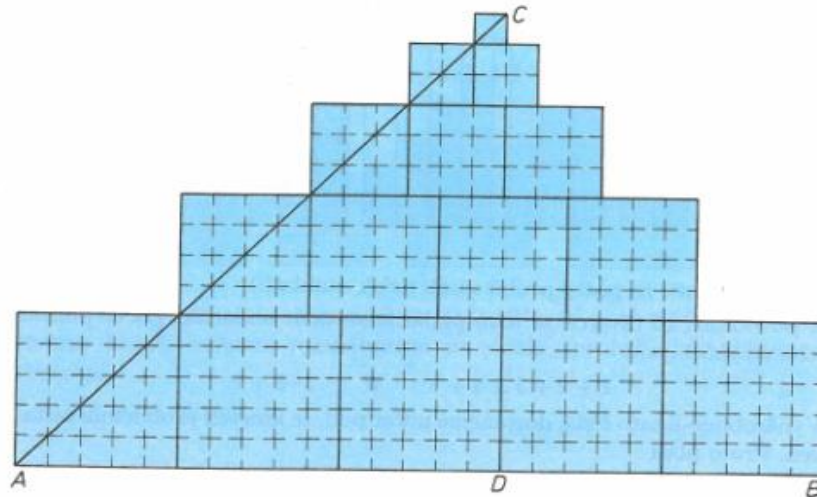University of South Bohemia
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age of pupils 12 - 13 years

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- Static visual proofs mostly do not capture the chain of thought leading to the proof, but only the result

- Additional dimension to visual proofs was brought by the use of DGS

Pythagoras' theorem (Perigal)

https://www.geogebratube.org/student/m135340

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While in the Pythagoras' theorem the dynamics was something extra,

which might help to better comprehension, in the proof of Euler´s formula

it is an integral part of the proof itself.

Euler's formula for polyhedra

https://www.geogebratube.org/student/m135341

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- Mathematics teachers, who are using computers (DGS) in their lessons

- They were asked to use dynamic visual proofs during their lessons

- Semistructured interviews with teachers focused on using of technologies and prooving

- Videostudies of mathematics lessons where dynamic visual proofs were used

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- Projector

- Students have their own computers

- Very difficult to organize and to force students to focus on proof
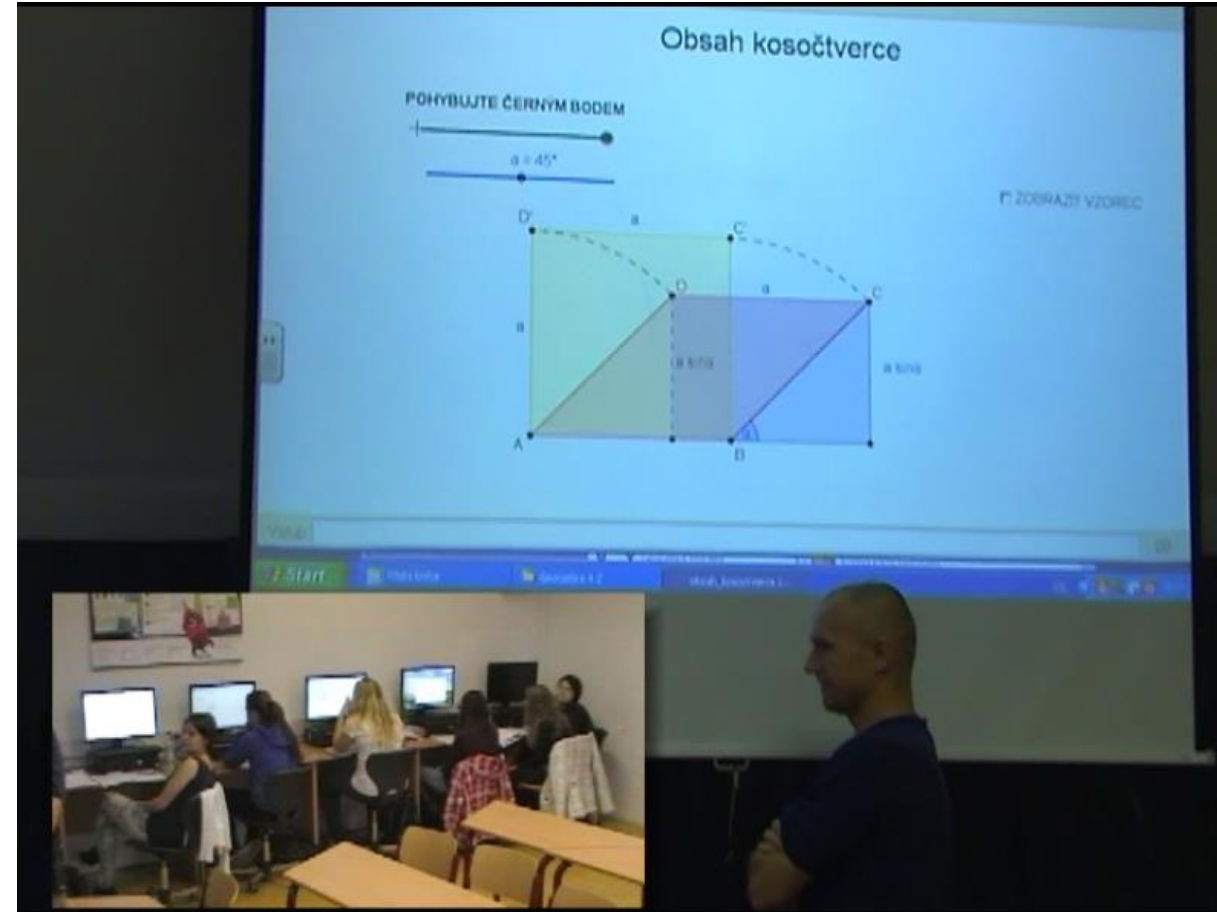
- Interactive whiteboard

- Students did not have their own computers

- Combination of dynamic visual proof and paper
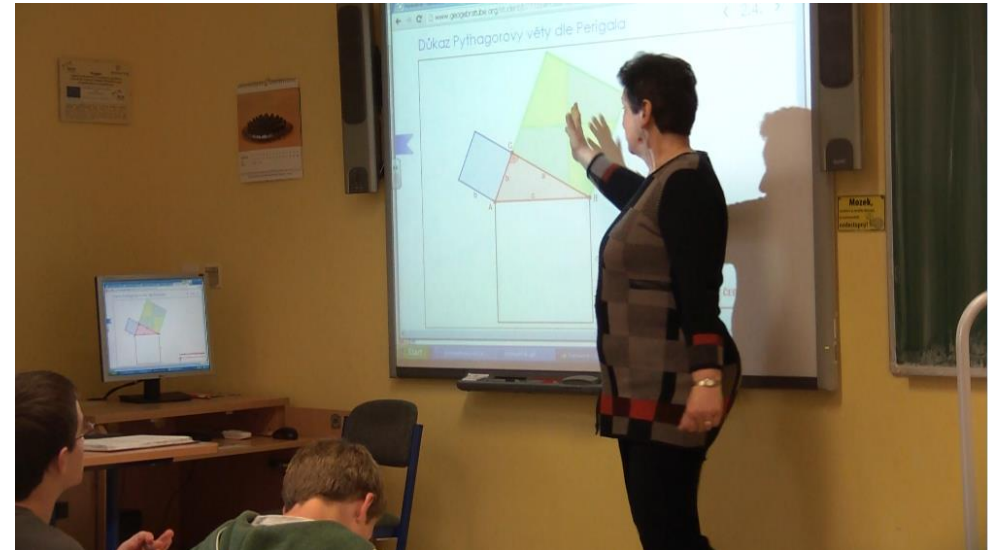
- A detailed analysis of the videos

- Reserach question: „How teachers use in the classroom dynamic visual proofs?"

- Recommendation for implementation visual proofs into teachers training program

- Popularization of visual proofs

BIBLIOGRAPHY

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AL], Jiří Herman ... [et]. *Matematika: Pro nižší třídy víceletých gymnázií : Výrazy 1*. 1. vyd. Praha: Prometheus, 1995, xii, 130 p. Classroom resource materials (Unnumbered). ISBN 978-807-1960-133.

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