Modern Mathematics lessons with technology and central school-leaving exam? Does this go together?

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Situation in Germany

The Federal Republic of Germany is a federalist state combine 16 federal states with 16 different school systems. Mathematics has at all high schools to be verified, however the number of hours at week, the teaching curriculum, contents and exam obligation are, considerably different from each other. So the lessons time is in the advanced level at high schools between 3 and 5 lessons per week. Mathematics is binding subject in the school-leaving exam in some countries. But one can replace the exam by a exam in another subject into other countries.

National education standards are developed therefore for the general matriculation standard in the subjects mathematics, German and English.

All countries are developing a pool of exam tasks, which will use the countries from 2017.

The countries Bavaria Brandenburg, Bremen, Hamburg, Lower-Saxony, Mecklenburg, Saxony and Schleswig-Holstein have since 2014 a common part of the central examination.

Within the last few years always more German federal states have decided in favor of the introduction of central examinations so that with the exception of Rhineland-Palatinate the mathematics examinations are carried out in all federal states with central tasks and in the respective country meanwhile at the same time.

Use of technology in the countries is very different. A pocket calculator is stipulated obligatorily with CAS in Thuringia (Since 1997). A graphic capable pocket calculator is obligatorily specified in Saxony, whether this CAS contains or doesn't decide the respective school. Graphic capable pocket calculators are in most other countries permitted with or without CAS but not stipulated obligatorily. Saxony-Anhalt permits only scientific pocket calculators without CAS.

Many federal states therefore differentiate the tasks at the central examination questions according to the kind of the authorized tools.

Situation in Saxony

There are central written checks in Saxony since 1992. The graphic capable pocket calculator must obligatorily be used in the lessons and in the examination since 1997. New teaching curriculums were introduced in 2004. These already describe the roll of the task culture in the preface in the mathematics lesson and demand that, mathematical tools like CAS, table calculation, function plotter and dynamic geometry software must employ. This means particularly that also pupils who use a graphic calculator without CAS had at least work on the computer with CAS in the lesson.

Statement from the didactic principles fixed in the teaching curriculum:

The mathematics lesson needs a task culture which the following kinds of tasks, used on an adequate scale:

- application-related tasks
- problem oriented tasks

- “open” tasks
- Tasks which connect basic contents from different fields of mathematics
- Tasks for select didactic strategy like
  - find differently solution ways
  - self organized study,
  - training of the language expression skill.
- Tasks with answer choice character (multiple Choice tasks)

Tasks which have to be worked with or without digital tools must be included in a balanced relationship.
Modern mathematics lessons means: didactically and medium didactically appropriately using of digital tools.

There had to be used:

- Tables and formulary without detailed classic examples
- Pocket calculator without graphic display (TR) as of class level 5,
- Pocket calculator with graphic display (GTR) as of class level 8
- mathematical software in the form of computer algebra systems (CAS) as of class level 8,
- dynamic geometry systems (DGS) and table calculation (TK)

Furthermore the new teaching curriculum in Saxony expels properly stringent, which mathematical competences must pupils had to know without tools.

Examples from the curricula in Saxony:

Class level 7:

- fundamental rules of arithmetic	
· mental arithmetic: tasks with easy numbers	$\left(-\frac{1}{2}\right) \pm \frac{3}{4};\ -0,2 \cdot (-0,1)$
· written: task with sing steps	$\left(-\frac{13}{6}\right) \pm \frac{11}{15};\ -3 : (-1,2)$
· with calculator: complex tasks	

Class level 11

- find derivation with and without CAS	proof some rules
· Derivation rules,	use CAS for discover the rules
· without CAS: Power functions with whole exponents, simply combinations with $f(x) = e^x$ or $f(x) = \sin x$	
- Solve equation systems	
· Without CAS: with simply numbers an with 2 or 3 variables	
· with GC or CAS: more than 3 variables	

At the specification of the degree of mastering an aid free arithmetic Saxony orientated himself at the article of Herget, Heugl, Kutzler and Lehmann which technical rake competences "are" indispensable in the CAS age?

The picture of mathematic lessons in public

You see in this picture a typical side from a German mathematics book. You perhaps cannot recognize everything. This isn't so bad. Simply enjoy the Eroticism of these terms. Imagine a 14-year teenager now:
He has so a "big interest" to experienced, what comes out at these terms and roots.

5 Mache den Nenner rational. a) $\frac{\sqrt{6}-\sqrt{2}}{\sqrt{2}}$ b) $\frac{1}{1+\sqrt{2}}$ c) $\frac{\sqrt{6}}{\sqrt{6}-\sqrt{5}}$ d) $\frac{\sqrt{24}+\sqrt{6}}{\sqrt{24}-\sqrt{6}}$

6 Bestimme auf 2 Dezimalen.
a) $\sqrt{5}+\sqrt{2}$ b) $\sqrt{5+2}$ c) $2+3\cdot\sqrt{2}$ d) $2\cdot\sqrt{5}-5\cdot\sqrt{2}$
e) $2\cdot\sqrt{3}-\sqrt{6}$ f) $7\cdot\sqrt{3}+\sqrt{10}$ g) $\sqrt{4\cdot 11-\sqrt{11}}$ h) $2\cdot\sqrt{63}-5\cdot\sqrt{28}$

7 Vereinfache soweit wie möglich.
a) $7\cdot\sqrt{3}+4\cdot\sqrt{3}$ b) $8\cdot\sqrt{2}-3\cdot\sqrt{2}$ c) $-2\cdot\sqrt{11}+2\cdot\sqrt{11}$ d) $\sqrt{10}-7\cdot\sqrt{10}+6\cdot\sqrt{10}$
e) $\frac{1}{2}\cdot\sqrt{2}+\frac{1}{3}\cdot\sqrt{2}$ f) $\sqrt{8}+\sqrt{2}$ g) $\sqrt{12}-\sqrt{3}$ h) $4\cdot\sqrt{50}-\sqrt{98}-\sqrt{18}$
i) $\sqrt{54}-2\frac{1}{3}\cdot\sqrt{6}$ j) $0,4\cdot\sqrt{3}+\frac{1}{4}\cdot\sqrt{48}$ k) $\frac{2}{3}\cdot\sqrt{45}-0,3\cdot\sqrt{20}$ l) $2\cdot\sqrt{112}+\sqrt{28}-\sqrt{252}$

8 a) $\sqrt{2}-\sqrt{3}+\sqrt{2}$ b) $2\cdot\sqrt{5}-3\cdot\sqrt{2}-\sqrt{5}$ c) $\frac{1}{2}\cdot\sqrt{10}+\sqrt{5}+\sqrt{10}+\sqrt{5}+4,5\cdot\sqrt{10}$
d) $\frac{\sqrt{64}}{3}+6\cdot\sqrt{3}-0.6\cdot\sqrt{175}+\sqrt{48}+\sqrt{7}$ e) $\sqrt{32}+\sqrt{8}-\sqrt{40}+\sqrt{242}-\sqrt{162}-\sqrt{80}$

9 a) $\sqrt{x}+\sqrt{x}-\frac{1}{2}\cdot\sqrt{x}$ b) $4\cdot\sqrt{y}+4\cdot\sqrt{z}-8\cdot\sqrt{z}$ c) $2\cdot\sqrt{a}+a\cdot\sqrt{2}-\sqrt{a}+a\cdot\sqrt{2}$
d) $\frac{3}{4}\cdot\sqrt{x}-0,8\cdot\sqrt{y}+\sqrt{0.25x}-\sqrt{\frac{y}{25}}-\sqrt{\frac{x}{16}}$ e) $15\cdot y\cdot\sqrt{x}-7y\cdot\sqrt{x}+8x\cdot\sqrt{y}+\sqrt{x^2y}$
f) $\sqrt{u^3}+\sqrt{v^3}-2.4\cdot u\cdot\sqrt{u}-\frac{1}{2}\cdot v\cdot\sqrt{v}-\sqrt{u^3}$ g) $s^2\sqrt{t}-0,6\cdot\sqrt{s^2t}+5,3s\cdot\sqrt{t}+0,7s\cdot\sqrt{s^2t}$

10 a) $(\sqrt{5}+\sqrt{3})^2$ b) $(\sqrt{10}-\sqrt{5})^2$ c) $(2\cdot\sqrt{3}+3\cdot\sqrt{2})^2$ d) $(8\cdot\sqrt{2}-2\cdot\sqrt{8})^2$
e) $(3\cdot\sqrt{5}-5\cdot\sqrt{3})(3\cdot\sqrt{5}+5\cdot\sqrt{3})$ f) $\sqrt{16-\sqrt{175}}\cdot\sqrt{16+\sqrt{175}}$

11 a) $(\sqrt{u}+\sqrt{v})^2$ b) $\sqrt{(p-q)^2}$ c) $(\sqrt{r}+\sqrt{s})(\sqrt{r}-\sqrt{s})$
d) $(\sqrt{2s}-5\cdot\sqrt{t})(\sqrt{2s}+5\cdot\sqrt{t})$ e) $(\sqrt{u+v}-\sqrt{u-v})(\sqrt{u+v}+\sqrt{u-v})$

12 Mache den Nenner rational. Gib einen Näherungswert an (3 Dezimalen).
a) $\frac{3\cdot\sqrt{5}+5\cdot\sqrt{3}}{\sqrt{6}}$ b) $\frac{2\cdot\sqrt{7}+\sqrt{18}}{2\cdot\sqrt{2}}$ c) $\frac{\sqrt{2}}{3+\sqrt{8}}$ d) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ e) $\frac{5\cdot\sqrt{3}-3\cdot\sqrt{5}}{5\cdot\sqrt{3}+3\cdot\sqrt{5}}$

13 a) $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ b) $\frac{5\cdot\sqrt{x}}{\sqrt{2x}-\sqrt{y}}$ c) $\frac{\sqrt{x+z}}{\sqrt{x}+\sqrt{z}}$ d) $\frac{2x-14}{7-\sqrt{7x}}$ e) $\frac{a-3}{3\sqrt{a}+\sqrt{3a}}$

We think about a gradual change of the assignment culture in Saxony at present.
You should know: Saxony has central school leaving examinations and graphic calculators are obligatorily specified.

Use of technology in the lessons

Tools like graphic calculator with and without CAS or computer be used for the support of solving tasks in Saxony in the lessons, primarily

- increasing the motivation of the pupils for mathematics,
- promoting understanding as regards content,
- discovering study and experiment make it possible,
- mathematical facts visualize to make more accessible to pupils with that,
- Tasks of solving on different ways and with different strategies,
- making realistic applications possible,
- combine various subjects of science and mathematics.

Use of technology in exam

At availability of the mentioned tools, since the exam 1999 Saxony stringently goes the way to allow these tools also in central exams.

Important is to test the mathematical competences purchased in the mathematics lesson. However, the main emphasis isn't any more algorithmically solving of routine tasks. Solving problems, modelling, and adequate dealing with the mathematical technical language and sure dealing with mathematical objects shall rather be in the center of the tasks.

So the classic curve discussions are replaced, descriptions, grounds and argumentations demanded by clothed, realistic tasks.

Tools are going by the pupils to use, but himself they are no exam object. So there is no questions about "sequences of buttons" etc.

Exam structure

I will show the development of assignment culture in examinations in the last years in three steps:

<u>An example for an exercise in a German school-leaving examination in 1994:</u>

There is given a function with the equation $y = f(x) = \dfrac{x^2 + 2x + 1}{4x - 4}$.

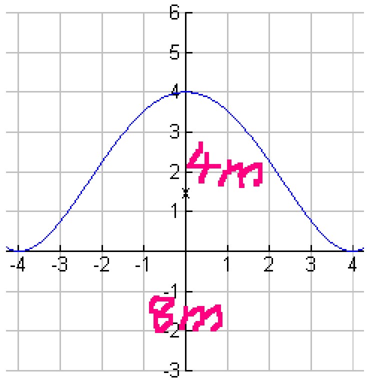
Discuss the characteristic attributes of the curve. Calculate the zeros, the coordinates of the intersection point with the y-axis and the coordinates of the local minimum points or maximum points.

A solution with the graphic calculator shows, that the discussion of curves is not up to date.

<u>Final secondary-school examination 1999:</u>

The symmetric gable of a baroque house is going to be reconstructed. The illustration shows the gable in a coordinate system. A symmetric polynomial function f describes the upper border of the gable. The x-axis is tangent at the graphs of the function f in the points (-4;0) and (4;0).

The height of the gable is 4 m.



a) Illustrate that the function f has to be at least of 4th order.

b) Define an equation of the function f.

c) The area of the gable is to be divided into two coextensive parts by a horizontal line. The upper part is to be decorated with ornaments, whereas windows are planed for the lower part .
Calculate the height the gable has to be divided at.

A ficticious example of an examination task:

Describe the shape of the gable with mathematical means.

Now the student had to write a mathematical essay and select his main emphases and mathematical tools himself

The written checks are bipartite in Saxony since 2010. In the first part the pupils work on relatively short tasks without every tools, only with paper and pencil. Those base competences which should be mastered without any tolls are checked. And there are on the one hand tasks to check the understanding as regards content.

To the clarification an example from the exam in 2012:

Exactly one answer is correct in the tasks 1.1 to 1.5 from the five out choices each. Mark the respective field with a cross.

1.1 Which term describes the firts derivation oft he function $f(x) = x^2 \cdot e^x \quad (x \in D_f)$?

☐	☐	☐	☐	☐
$2 \cdot x \cdot e^x + e \cdot x^2$	$x^2 \cdot e^x$	$2 \cdot x \cdot e^x + x^2$	$2 \cdot x \cdot e^x$	$2 \cdot x \cdot e^x + x^2 \cdot e^x$

1.2 The function f mit $y = f(x) = \dfrac{x^2 - x}{x - 1} \quad (x \in D_f)$ have on $x = 1$

- ☐ the value $y = 1$
- ☐ no value
- ☐ a root
- ☐ a pole
- ☐ a minimum

1.3 which integral with $a \in \mathbb{R}$ und $a > 0$ have the value 0?

☐	☐	☐	☐	☐
$\int_{-2}^{2}(x+a)\,dx$	$\int_{-2}^{2}e^{a\cdot x}\,dx$	$a\cdot\int_{-2}^{2}\sin x\,dx$	$a\cdot\int_{-2}^{2}x^4\,dx$	$\int_{-a}^{a}(x^2+2)\,dx$

1.4 The line g with $g: \vec{x} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}$ $(t \in \mathbb{R})$ goes

- ☐ parallel to x-y-plane
- ☐ parallel to y-z-plane
- ☐ parallel to x-z-plane
- ☐ parallel to z-axis
- ☐ through O(0;0)

1.5 There is given a rational function f with $D_f = \{x \mid x \in \mathbb{R}\}$, The first derivation f´has the following qualities:

(1) f' have exactly one root.
(2) For all $x \in \mathbb{R}$ is: $f'(x) \le 0$.

What is true?

- ☐ the function f have one pole.
- ☐ The function f have one minimum.
- ☐ The function f have one maximum.
- ☐ The function f is increasing.
- ☐ The function f have one turning point.

2 There is given a plane E with $E: x + 2\cdot y - 2\cdot z = 2$ and for all a $(a \in \mathbb{R})$ a plane G_a with $G_a: 3\cdot x + 4\cdot y + a\cdot z = 1$.

For all b $(b \in \mathbb{R})$ is given a point $P_b\left(1 \middle| -2 \middle| b\right)$.

2.1 Find the value b, that the point P_b lies in the plane E..

2.2 Find the distance of the points P_b from the plane G_a for $a = 0$.

2.3 Mention the value a, that the planes E and G_a are orthogonal.

4 There was a survey with 32 pupils.

Exactly 20 pupils have an computer, exactly 14 pupils have a bicycle and exactly 4 pupils haven´t a computer and haven´t a bicycle.

Find the probability that any pupil have a computer and also a bicycle.

<u>(end oft he example)</u>

Some few multiple Choice tasks are in these task parts. A task construction with the help of so-called 0-1 items declines Saxony here, in principle, since she doesn't meet the requirements to a check to the proof of studying ability.

Application tasks at which pupils must if possible use acquired abilities in unknown contexts are then provided in the second part of the check.

<u>Parts of a task from the exam 2012 in Saxony (tools: Graphic Calculator with or without CAS)</u>

Task B 1

For construction of a street a bridge shall be set up over a valley which is flowed through by a creek.
The profile line of the valley below the planned bridge can be descripted in a Cartesian coordinate system (1 length unit corresponds to 1 meter) approaching by the graph of the function f with the equation:

$$y = f(x) = \frac{1}{10\,000\,000} \cdot x^4 + \frac{7}{200\,000} \cdot x^3 + \frac{1}{400} \cdot x^2 + 20 \quad (x \in \mathbb{R}; -300{,}0 \le x \le 100{,}0)$$

The x-axis goes along the horizontal lines. The profile line of the road of the bridge shall connect the points A(-300;f(-300)) und B(100;f(100)).

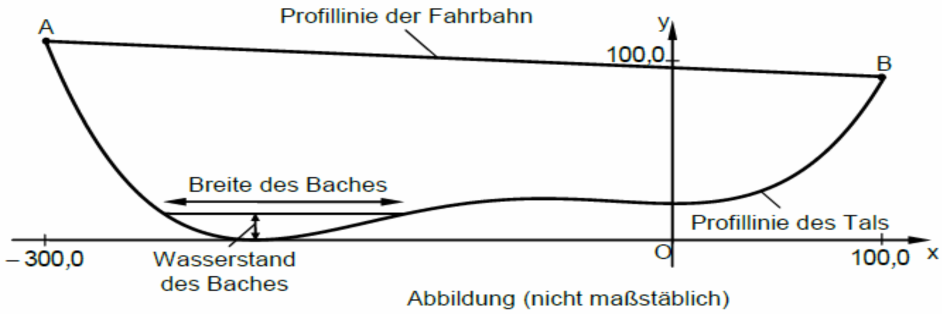


Abbildung (nicht maßstäblich)

1.1 Indicate the coordinates the deepest point the profile line the valley. In the point P(0;f(0)) is a measurement point. Say at how many meters the measurement pont is over the deepest point of the profile line of the valley.

1.2 Determination the length of the profile line of the road of the bridge.

…

1.7 The planners of the bridge hold the opinion, that 2.5% of the animals living in the environment of the planned bridge are protected.
Animal conservationists suspect, however, that this quota is 10.0%.
These opinions shall be tested. To this the number of protected animals shall be determined from exactly 80 animals sighted in the surroundings of the planned bridge. Becomes the acceptance of the planners of the bridge as a null hypothesis and the acceptance of the animal conservationists regards as an alternative hypothesis. At The rejection area of the null hypothesis shoud be A=(4;…, 80).
Show that for this rejection area the probability for an error of 1th type is more than 5%. .
Find the probability for this rejection area for an error of 2nd type.

…
(end oft he example)

Important is here the combination of various fields of mathematics. In the exams in sxony You find no tasks which are titled with “Analysis” or “Algebra”.

There are no different tasks for students using a graphic calculator without CAS or with CAS. In the documentation of the solving process You can see, if students solve an equation with graphic tools or with algebra tools.

The way of the solution had to be comprehensible.

Outlook

In the future there will be the questions:
How can You check competences like problem solving or to argue in the central written exams?
How can we convert exams in teams?

There are reservations because of traditions in the countries just as juridical problems.

Conceivable are practical parts of the exams, where students solve a mathematical problem in a team. The teacher should observe and document the working process oft he team.

Literature:

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