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Mathematical Amazements and Surprises on Pi-Day

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Abstract

It is all too often the case that we present concepts in mathematics without enriching students by exposing them to the concepts' proper background. This is the case with the ubiquitous number represented by the Greek letter π. In the United States dates are written in the order of month, day, and year. Therefore, on March 14 each year schools across the United States celebrate π day.

Keywords:
Pi-Day
Approximations of Pi

Some teachers add a little spark to the day by mentioning that that date happens also to be Albert Einstein's birthday. However, in the year 2015 the mathematics community took special ride in pointing out that March 14 was a particularly appropriate day for π as illustrated below.

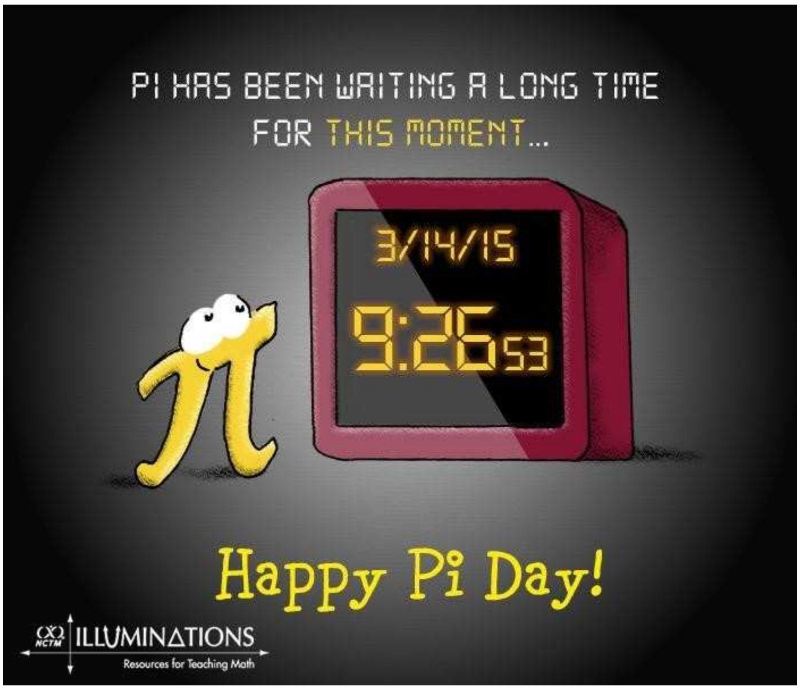

Illustration 1: Happy π Day

As you can see this is the ultimate special π day, since we have the time marked as 3.14-15 9:26:53, which represents π correct to nine places. Here is the value of π to many more places,

3.1415926535897932384626433832... .

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If you wish to find the value of π to many hundreds of places, please go to the Karlsplatz Opernpassage in Vienna and you will find it inscribed on the mirrored wall many many meters long. It is important to realize that the expansion of the value of π will go on to infinity, even though this incredibly long list of places on the wall is impressive one needs to bear in mind that we already have calculated this value into the trillions of places.

We often seek entertainment in mathematics and we encourage teachers to do the same in their teaching. Perhaps the most famous American in history is the first president, George Washington, who was born on February 22, 1732. If we write his date in simple numeral form it would appear as the string **02221732**, which occurs at position 9,039,149 of the infinite expansion of the value of π. Through the use of computers, we can show that this string occurs 3 times in the first 200,000,000 digits of π, counting from the first digit after the decimal point. (The 3 is not counted.) Here is the string and the surrounding digits of the first occurrence of this string of digits:

3.14159265...47814901349324297418**02221732**49756451826 61284136

The question often comes up as to where did this symbol originate, which represents the ratio of the circumference of a circle to the diameter? In 1706, William Jones (1675 – 1749) published his book, *Synopsis Palmariorum Matheseos*, where he used π to represent the ratio of the circumference of a circle to its diameter. In 1736, Leonhard Euler began using π to represent the ratio of the circumference of a circle to its diameter. But not until he used the symbol π in 1748 in his famous book *Introductio in analysin infinitorum* did the use of π to represent the ratio of the circumference of a circle to its diameter become widespread.

Let us now take a look at the value of π from a geometric point of view. In figure 1 we notice regular polygons inscribed in a circle of increasing number of sides moving from left to right. As a number of sides of the polygons increase the perimeter of the polygons begin to approach the circumference of the circle. In the center diagram, we have the value of the angle *x* as: $\angle x = \frac{1}{2} \cdot \frac{360°}{n} = \frac{180°}{n}$, we can then take $\sin \angle x = \frac{a}{1/2} = 2a$, and then get one side length to be equal to $\sin \frac{180°}{n} = 2a.$ This allows us to find the perimeter of an *n*-sided regular polygon to be equal to $n \sin \frac{180°}{n}$. Therefore $\lim_{n \to \infty} n\ \sin \frac{180°}{n} = \pi.$

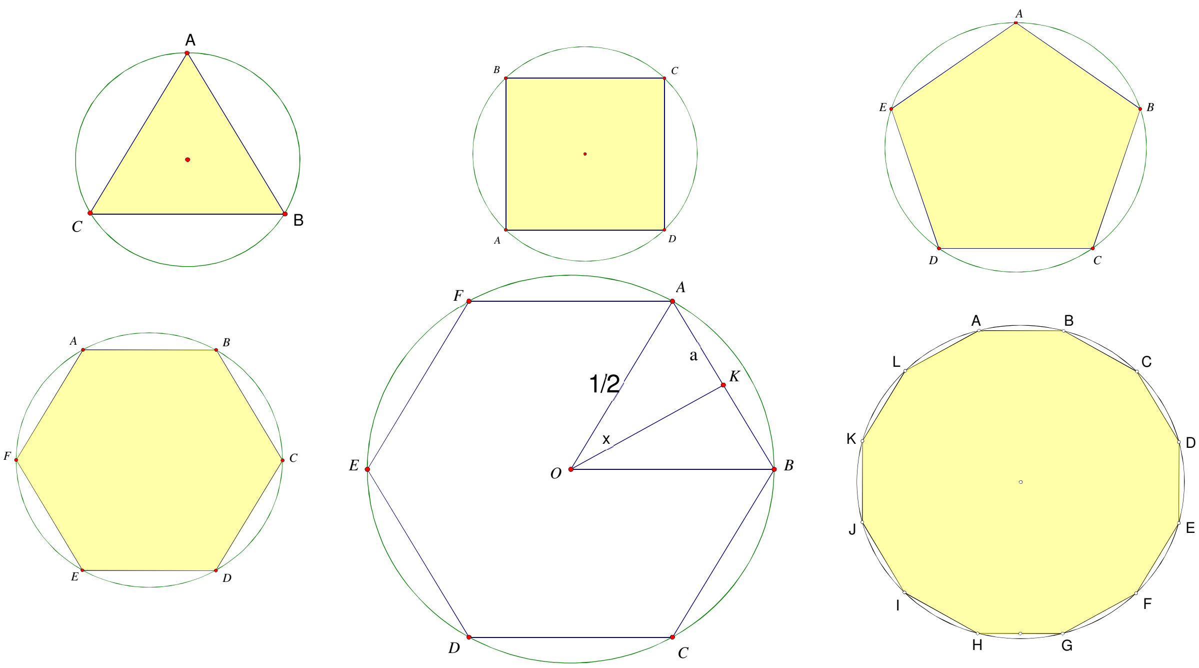


Fig. 1: Approximation of a circle by polygons

Therefore, we can see that as the number of sides increases for the inscribed polygons, as well as for circumscribed polygons, as you can see from the table below, the perimeter of each approaches the value of π, where the diameter has length 1.

n	Perimeter of inscribed polygon of *n* sides	Perimeter of circumscribed polygon of *n* sides
3	2.5980762113533159402911695122588…	5.1961524227066318805823390245176…
4	2.8284271247461900976033774484194…	4.000000000000000000000000000000…
5	2.9389262614623656458435297731954…	3.6327126400268044294773337874031…
6	3.000000000000000000000000000000…	3.4641016151377545870548926830117…
7	3.0371861738229068433303783299385…	3.3710223316527005103251364713988…
8	3.0614674589207181738276798722432…	3.3137084989847603904135097936776…
9	3.0781812899310185973968965321403…	3.2757321083958212521594309449915…
10	3.0901699437494742410229341718282…	3.2491969623290632615587141221513…
11	3.0990581252557266748255970688128…	3.2298914223220338542066829685944…
12	3.1058285412302491481867860514886…	3.2153903091734724776706439019295…
13	3.1111036357382509729337984413828…	3.2042122194157076473003149216291…
14	3.1152930753884016600446359029551…	3.1954086414620991330865590688542…
15	3.1186753622663900565261342660769…	3.1883484250503318788938749085512…
24	3.1326286132812381971617494694917…	3.1596599420975004833166349778332…
36	3.137606738915694248090313750149…	3.1495918869332641879926720996586…
54	3.1398207611656947410923929097419…	3.1451418433791039391493421086004…
72	3.140595890304191984286221559116…	3.1435878894128684595626030399174…
90	3.14095470322508744813956634628…	3.1428692542572957450362363196353…
120	3.1412337969447783132734022664935…	3.1423105883024314667236592753428…
180	3.1414331587110323074954161329369…	3.141911687079165437723201139551…
250	3.1415099708381519785686472871987…	3.1417580308448944353707690613384…
500	3.1415719827794756248676550789799…	3.1416339959448860645952957694732…
1,000	3.1415874858795633519332270354959…	3.1416029890561561260413432901054…
10,0000	3.141592601912665692979346479289…	3.1415927569440529197246707719118…
π	3.141592653589793238462643383279502….	3.141592653589793238462643383279502….

Table 1: Perimeters of the inscribed and circumscribed polygons

All too often students are simply given the formula for the area of a circle without any justification as to how it may be justified. Here is a simple way to justify the formula for the area of a circle.

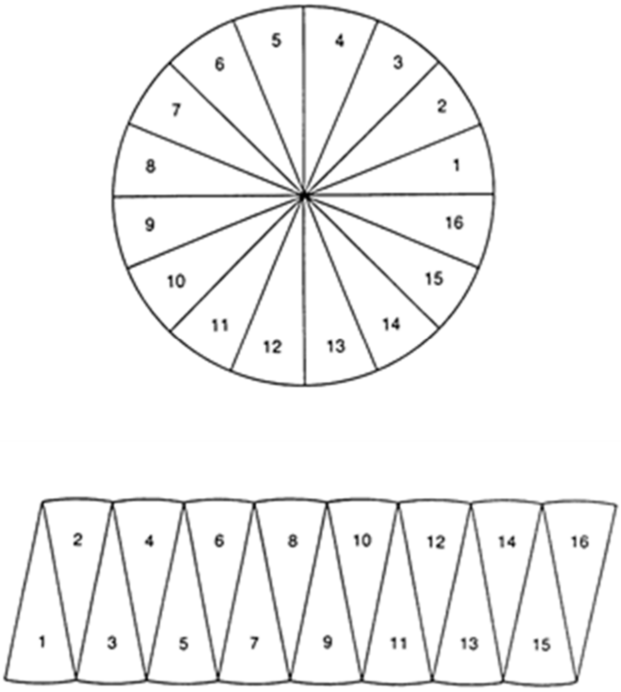


Fig. 2: Area of the circle

Suppose we divide a circle into a large number sectors as shown in figure 2. In this case, we are using 16 sectors so that it can be easily seen. We then take the sectors apart as shown in the lower part of figure 2. The figure that we have formed looks like a parallelogram, and when we have a huge number of sectors instead of just the 16 we show in the figure, it would look just like a rectangle. The base of the parallelogram has half the length of the circumference of the circle, or πr, while the height of the parallelogram is equal to the radius of the circle, or r. The area of the parallelogram is equal to the length of the base times the length of the height, which is

$$Area = \pi r \cdot r = \pi r^2,$$

which is our well-known formula for the area of the circle.

Suppose we now consider a circle inscribed in a square, and compare the areas the circle whose radius is r, and therefore, the side of the square has length $2r$, as shown in the figure 3.

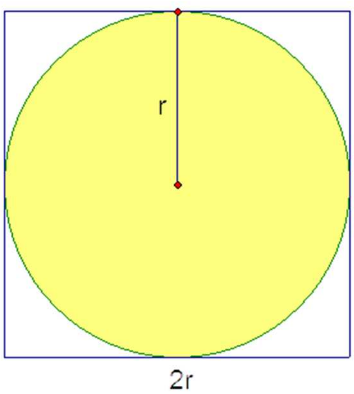


Fig. 3: Circle inscribed in a square

We can represent the ratio of the areas as:

$$\frac{Area_{square}}{Area_{circle}} = \frac{(2r)^2}{\pi r^2} = \frac{4r^2}{\pi r^2} = \frac{4}{\pi} \approx 1.273239$$

which then allows us to get the following value π:

$$\pi \approx \frac{4}{1.273239} \approx 3.141594$$

There are times when rather silly things crop up in the political realm. This was precisely the case on January 18, 1897 in the state of Indiana. A legislator Edward J Goodwin introduced a bill into the Indiana legislature as follows:

"A bill for an act introducing a new mathematical truth and offered as a contribution to education to be used only by the State of Indiana free of cost by paying any royalties whatever on the same, provided it is accepted by the official action of the legislature of 1897."

If this were taken seriously, his values for π could have been any of the following: 4, or 3.160494, or 3.232488, or 3.265306, or 3.2, or 3.333333, or 3.265986, or 2.56, or 3.555556. Fortunately, the silliness of his ways was disposed rather quickly and nothing ever came of it, but there are always attempts at redefining the value of π. Having written a book, *"π: A Biography of the World's Most Mysterious Number,"* which is now been translated into about a dozen languages, I still get mail from various parts of the world from readers who feel that they had come up with a new value of π, something we now know that is totally ridiculous.

Where today we have the value of π calculated to over one trillion places, we often wonder what was the value of π in ancient times? Until an article I published in the *Mathematic Teacher Journal* in January 1984, most history of mathematics books believed that the most primitive value of π, was that described in the Old Testament of the Bible. In this article we reported that we discovered that in the late 18th century the Rabbi Elijah of Vilna (1720 – 1797), who was also a mathematician, made a very interesting discovery using a time-tested technique to analyze the biblical scriptures in the Hebrew language. This technique is known as gematria. He found that there were two almost identical sentences describing a circular fountain in King Solomon's temple courtyard. The description appears once in 1 Kings 7:23, and another time in 2 Chronicles 4:2. In translated form the sentence reads:

"And he made the molten sea of ten cubits from brim to brim, round in compass, and the height thereof was five cubits; and a ***line*** *of thirty cubits did compass it round about."*

From this sentence, we would calculate the value of

$$\pi = \frac{30}{10} = 3.$$

However, he noticed that the word for line measure was spelled differently in both sentences, despite the fact that all the other words were spelled exactly the same in both sentences. The process of gematria is one where the letters of the Hebrew alphabet are also used to describe numbers.

In 1 Kings 7:23 the word is spelled: קוה

In 2 chronicles 4:2 the word is spelled קו

Using gematria, the individual letters have the following numerical values: ק = 100, ו = 6, and ה =5.

In Kings the word for line measure has value 5 + 6 + 100 = 111, while in Chronicles the word for line measure has value 6 + 100 =106. Taking the process of gematria one step further, the quotient of these two numbers is

$$\frac{111}{106} = 1.0472.$$

Here is the part that really strikes amazement. When we multiply this number, 1.0472, by the value that we read in the Scriptures about the size of the pool, where we get the value for π equal to 3, and then multiply this by 1.0472, we get 3.1416, which is a correct value of π to 4 places and simply unheard of in that time. Is this mere coincidence? We leave that to the reader to ponder.

By the way, the famous German artist and mathematician, Albrecht Dürer (1471 – 1528), used an approximation for π of $\pi = 3\,\frac{1}{8} = 3.125$. A brief history of the development of the value of π can be seen in the following table:

Who calculated	When	Number of decimal place accuracy	Value found
Babylonians	2000? BCE	1	3.125 = 3 + 1/8
Egyptians	2000? BCE	1	3.16049 = $(\frac{16}{9})^2$
Bible (1 Kings 7:23)	550? BCE	1 (4)	3 (3.1416)
Archimedes	250? BCE	3	3.1418
Vitruvius	15 BCE	1	3.125
Ptolemy	150	3	3.14166
Liu Hui	263	5	3.14159
Tsu Ch'ung Chi	480?	7	3.1415926 $=\frac{335}{113}$
Brahmagupta	640?	1	3.162277=$\sqrt{10}$
Al-Khowarizmi	800	4	3.1416
Fibonacci	1220	3	3.141818181818204667768595…
Viete	1593	9	3.1415926536
Romanus	1593	15	3.141592653589793
Van Ceulen	1615	35	3.1415926535897932384626433832795029
Newton	1665	16	3.1415926535897932

Table 2: Historical values for the approximation of π

Year	Mathematician	Number of place accuracy of π	Time for calculation
1954	S. C. Nicholson & J. Jeenel	3,092	13 minutes
1954	G. E. Felton	7,840	33 hours (Generated 10,021 places but only 7,480 were correct due to machine error.)
1958	François Genuys	10,000	100 minutes
1959	Jean Guilloud	16,167	4 hours, 20 minutes
1961	Daniel Shanks & John W. Wrench, Jr.	100,265	8 hours, 43 minutes
1966	M. Jean Goilloud & J. Filliatre	250,000	41 hours, 55 minutes
1967	M. Jean Goilloud & Michele Dichampt	500,000	44 hours, 45 minutes
1973	M. Jean Goilloud & Martine Bouyer	1,001,250	23 hours, 18 minutes
1981	Kazunori Miyoshi & Kazuhika Nakayama	2,000,036	137 hours, 20 minutes
1982	Yoshiaki Tamura & Yasumasa Kanada	8,388,576	6 hours, 48 minutes
1982	Yoshiaki Tamura & Yasumasa Kanada	16,777,206	Less than 30 hours
1988	Yoshiaki Tamura & Yasumasa Kanada	201,326,551	About 6 hours
1989	Gregory V. & David V. Chudnovsky	1,011,196,691	Not known
1992	Gregory V. & David V. Chudnovsky	2,260,321,336	Not known
1994	Gregory V. & David V. Chudnovsky	4,044,000,000	Not known
1995	Takahashi & Yasumasa Kanada	6,442,450,938	Not known
1997	Takahashi & Yasumasa Kanada	51,539,600,000	About 29 hours
1999	Takahashi & Yasumasa Kanada	206,158,430,000	Not known
2002	Yasumasa Kanada	1,241,100,000,000	About 600 hours
2013	Shigeru Kondo	12,100,000,000,000	90 days
2014	Houkounchi	13,300,000,000,000	Not known
2016	Peter Trueb	22,459,157,718,361	105 days

Table 3: Contemporary values for the approximation of π

If your wonderment has not been stretched till now, here is one that will surely get you to "wonder." Imagine that we can obtain a reasonably good approximation of π by simply dropping a needle on a lined piece of paper. This is what the French naturalist, Georges Louis Leclerc, Comte de Buffon (1707 – 1788) espoused with the following activity:

Take a sheet of paper with equally spaced ruled lines, and a needle of length equal to the distance between the lines. The probability that the needle will touch one of the lines is

$$P = \frac{Number\ of\ line\text{-}touching\ tosses}{Number\ of\ all\ tosses}.$$

He claimed that to calculate the value of π we need to toss the needle (equal in length to the space between the lines) and tallying the line touching tosses and the total number of tosses. Then place these into the following formula:

$$\pi = \frac{2 \times number\ of\ all\ tosses}{number\ of\ intersection\ tosses},$$

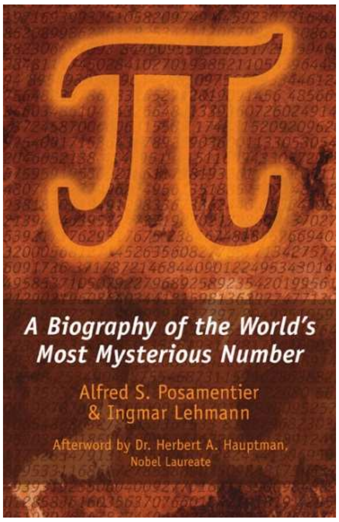
which should be reasonably accurate. In 1901 the Italian mathematician Lazzerini tried this experiment with 3408 tosses of the needle. He arrived at the fraction

$$\frac{355}{113}=3.14159292035398230088495575221 24\ldots,$$

which is extremely close to the actual value of π to this many decimal places.

There are endless discussions we can have about this most ubiquitous ratio of the circumference of a circle to its diameter, which we call π. Interested readers may wish to access a book which will take you on a far more extensive journey than we can do in this very brief presentation.

The book is: *"π: A Biography of the World's Most Mysterious Number"* by Alfred S. Posamentier and Ingmar Lehmann (Prometheus Books, 2004).



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