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School Mathematics and Mathematical Training: Two Hotspots in the Curriculum Development for Teacher Education

TSG 37: Mathematics curriculum development

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Abstract

In Austria a paradigmatic change actually happens: for specific subjects at secondary school level a common and unique education for all student teachers will be introduced in the next future. Even for a rough estimation of this development it is important to know that there are still at least two quite different school types at secondary level 1 (concerning pupils from 10 to 14). A complex process of discussion therefore happens. We highlight (also empirically) two essential aspects of this issue.

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1 Austrian School System – A short overview

After four years of primary education pupils have to choose between the New Secondary School (NMS) and the Academic Secondary School Lower Level (AHS Lower Level). More than 60% of the pupils who pass AHS Lower Level continue their education at secondary level 2 to get the higher education entrance qualification. More than 50% of the persons leaving a NMS decide to attend Schools for Intermediate Vocational Education (BMS) or Colleges for Higher Vocational Education (BHS) (data recorded by http://www.statistik.at for 2013/2014). These different types of school are considered by separated teacher educations. On the one hand there is the Pädagogische Hochschule (PH, Pedagogical University) which provided formerly the education of NMS teachers and on the other hand the University is designated for the education of future AHS respectively BHS teachers. As a consequence the PHs focus on pedagogical contents in their teacher education and the universities concentrate on pure mathematical subjects (pars pro toto) without connections to school practice.

2 Looking for a common curriculum

For developing common curricula concerning all future secondary level teachers the already implemented bachelor-master system has to be regarded. One has to study two subjects. An Austrian specific feature is that the bachelor studies take at least four years (240 ECTS, cf. Table 1 which is from the University of Vienna).

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Scientific discipline Subject 1: 70-80 ECTS	Subject 2: 70-80 ECTS	**Fundamental principles of pedagogic**: 34 ECTS 19 of these dedicated to school practice
Domain of free choice: 10 ECTS		
Subject didactics Subject 1: 15-25 ECTS	Subject 2: 15-25 ECTS	**School practice**: 6 ECTS

Table 1: Curriculum of the Bachelor Program in Secondary Teacher Education.

Four clusters between PHs and universities were founded to prepare the implementation. They cover different parts of the country. We are both members of the mathematical subgroup of cluster east (including four PHs and one university) belonging to different institutions.

In these new designed studies the mathematical education is totally separated from the study of mathematics (for those students who are not becoming teachers but mathematicians). This is based on the assumption that student teachers in mathematics have specific needs concerning their mathematical knowledge. These needs must initially be realized and then the findings have to result in a conception to satisfy students' requirements.

Another point is that the tradition of so-called school mathematics lectures which cover mathematical aspects as well as elements of subject didactics (compare Felix Klein: "Elementary mathematics from an advanced point of view.") could also be a potential link between the two different cultures (University, PH). But in detail the curriculum discussion shows that quite different beliefs in possible achievements of school mathematics lectures exist. To fulfil the purpose mentioned above it is also necessary to analyse accurately school mathematics lectures.

2.1 Research method

We have planned interviews with experts to investigate both – the nature of school mathematic lectures and the education of student teachers in mathematics (in opposite to the education of future mathematicians). We studied the curricula of different universities in Austria with respect to our focus to choose these experts. This means looking for specific lectures for student teachers in mathematics (which is stressed in their titles) and courses which are called something with "school mathematics". This analysis leads to a dozen persons who are working at the Universities of Vienna, Linz and Graz. Four of them refused so that we finally interviewed eight colleagues in 2015, half of them on school mathematics (S1 – S4) and the others on student teachers' mathematics education (M1 – M4). For that reason we create two different structured interview guides. Afterwards we transcribed the recorded interviews and evaluated them using the Grounded Theory (Glaser & Strauss 2010).

2.2 Mathematics Education of student teachers

Asking for differences between the mathematical education of student teachers and of students in mathematics the most important aspect stresses the different ways of the continuation and the different goals. In opposite to the lectures of the mathematics students the student teachers are contented with only one course covering a specific mathematical field, e.g. calculus or linear algebra. In those lectures mathematics students become familiar with techniques ("tricks") to calculate a certain limit for instance and they train these specific methods for solving also sophisticated problems. Quite different to this claim the interviewed university lecturers see the need of student teachers to seek out the relevance of certain passages in the mathematical courses for their future jobs. For instance in the lecture on numerical mathematics for student teachers the question how does a hand-held calculator calculate the sine of 47° is discussed (M2). Another example is to point out the mathematical fundament of sketching curves (M1).

Another issue in our survey is the handling of proofs in student teachers courses. The common opinion of all experts is that proving is the most important part in every mathematical lecture. But there are two quite different positions on this topic depending on the point of time when the theorem which has to be proved occurs. Some say that at the beginning of their courses they teach very closely to the deductive character of mathematics without gaps in their argumentation: mathematics as "*well-oiled machine*" (M1). Otherwise due to the time-limited frame of the courses without the possibility to continue in another course the lecturers are often forced to become narrative. This means that they tell at the end only the fundamental ideas without proving exactly: The Jordan canonical form in a linear algebra course for instance is only "*represented to show what it looks like and to get an idea without proof of it*" (M4).

During the development process of the curricula the involved mathematicians often declare that it is important to show an unbiased image of the science of mathematics. All interviewed persons correspond to the intention to introduce the student teachers to recent research activities. M1 realizes this purpose in discussing elementary deep mathematical problems such as the four color theorem. Another way is to demonstrate modern applications of mathematical methods. For that reason a "*critical mass*" (M2) of mathematical tools is necessary to be gained by the student teachers. M3 offers a third possibility: students become familiar with how mathematicians work. Last but not least M4 emphasizes the opportunity to listen to talks of famous mathematicians in the mathematical colloquium which is established at almost every university.

It is a widespread impression that student teachers are less motivated to learn mathematics at university level, because they think it is irrelevant for their future career as teachers at schools (e.g. Beutelspacher et al. 2011). All interviewed mathematicians conform that there is no general solution for this problem. Furthermore, economic reasons are crucial for students' motivation (M2). One mathematician even reports that the beginners' courses have also "*selection purposes*" (M3). This is an intended side effect of the axiomatic way to start these lectures: "*we don't want people to become teachers if they are not able to pass these [courses]*" is a frequently mentioned opinion at mathematical departments as M3 refers without being in agreement with this opinion.

2.3 School Mathematics – what does it mean?

The missing link between the two hotspots mentioned in the title of this paper could be the identification of the required background knowledge in mathematics, which every teacher needs to teach mathematics soundly at schools (e.g. Usiskin et al. 2003). It is important to cope with students' statements on mathematical questions like how to prove the intercept theorems in the irrational case (S2). Moreover, we illustrate another aspect with an example. There are two important theorems for calculus as it is treated in school: the well-known sufficient condition for local extrema of differentiable functions and the monotonicity theorem (the connection between the sign of the derivation and the monotonicity of a function). In opposite to the analysis at university the mean value theorem does not matter usual instructions in calculus. "*You have to know as a teacher: what are the absolutely central theorems that I need at school?*" (S2).

Another point in common is building bridges between mathematics which is taught at universities and mathematics at schools. In detail there are little but essential differences. First it is important to link calculus school mathematics with analysis, "*because many theorems of analysis are used unproven at school; to make clear that these are provable statements*" (S1). But it's also the case to ask: "*which offspring from the science mathematics have their flowers in the mathematical lessons at school?*" (S2). Another point of view is "*to break down*" scientific contents to school matters (S3, e.g. Götz & Süss-Stepancik 2013). A fourth approach is to discuss the development of a certain conception (the (surface) area) from primary school (rectangle) to university level (Riemann integral): "*Integral is more than area […] coming away from the misconception: calculation of areas is integral.*" (S4).

Citing examples from analysis in respect to build bridges is not entirely coincidental. Other contents like arithmetic and algebra implicate different approaches: The students are already familiar with calculation techniques from school and therefore the semantic level has priority (S1). School mathematics also fills gaps where no counterpart in university mathematics is given: percent calculations, directly and inversely proportional relations, scale or construction with compass and ruler, for instance (S3).

3 Consequences for curricula development

In our new curriculum one starts with issues of elementary geometry like intercept theorems, the theorem of Pythagoras, inscribed angle theorem and consequences for the geometry of triangles to bridge the gap between school and university mathematics. The intention behind this paradigm change (away from analysis and linear algebra at the beginning) is to make the students more sustainably familiar with the typical sequence definition – theorem – proof – example. For two reasons we believe this measure leads to success (a necessary evaluation is still open because this curriculum became applicable 2014/15): first the possibility to illustrate obtained results and last but not least the motivational aspect for student teachers. They deal with material which they will use as teachers and that they may know from their own school experience.

Another beginner course discusses various aspects of mathematics (epistemological, historical and philosophical aspects, applications, ideas of subject didactics) at a low level to offer the student teachers an overview about their future subject. This lecture is also intended to give the students a little impression of the science mathematics.

The main consequence for student teachers' education is the schedule of school mathematics lectures always right after the related courses in mathematics in two immediately consecutive terms. So the possibility occurs to soon make clear the relevance of certain mathematical topics also for school education and to add some important contents for student teachers like formulas for the volumes of solid figures, which are not discussed in the geometry lecture mentioned above.

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