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The use of dynamic worksheets to support functional thinking in lower secondary school

TSG 42: Uses of technology in lower secondary mathematics education (age 10 to 14)

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Abstract

In teaching, a variety of difficulties arise when working with functions. Therefore, the question arises what is the influence of technology-based material on the individual conceptions of students of lower secondary school. Are dynamic worksheets able to help students to develop appropriate conceptions? In this talk, specific dynamic GeoGebra applets which are intended to support the development of appropriate skills as well as the research design and results of a corresponding qualitative study of a PhD project are presented.

Keywords:
Educational research
Technology
Functional thinking
Lower secondary school

1 Problem description

In the literature various difficulties of students in the context of functional dependencies are described. Especially the following are prominent:

Poorly developed co-variational aspect
The co-variational aspect of functional thinking is very important for working with functions in practice. But empirical studies show that especially this aspect is underdeveloped with students (De Bock, Verschaffel, & Janssens, 1998; Malle, 2000; Hoffkamp, 2011).

Graph-as-picture error
A poorly developed co-variational aspect can be seen, among other things, on the graph-as-picture error. This occurs in various forms and means that students see functional graphs as a photographic image of a real situation (Clement, 1989; Schlöglhofer, 2000; Hoffkamp, 2011). Through an intensive dealing with such tasks the understanding of graphical representations can be deepened.

Illusion of linearity
Another problem of students is the so called "Illusion-of-linearity". This means that linear or directly proportional models are preferably used for the description of relations (De Bock, Van Dooren, Janssens, & Verschaffel, 2002; Hoffkamp, 2011).

Leading to the use of technology
Dynamic mathematics software like GeoGebra is suitable to emphasize the co-variational aspect of functions through its interactive representations and therefore may support the development of functional thinking (Hohenwarter, 2006). I have created several GeoGebra applets considering different design criteria (Clark &

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Mayer, 2011). They address to the transfer between situational model and graph. The following dynamic worksheet is based on a task of Schlöglhofer (2000).

This applet simulates the following situation: From the point P a red billiard ball is shot along the specified path. Here d is the distance of the ball from the top of the billiard table, it is a function of time. The applet contains a model of the situation and a graphics view with a so-called Dynagraph representation, which uses two parallel axes instead of two orthogonal. On the lower axis the argument t and on the upper the associated function value d(t) is shown. By playing the animation or using the slider the associated function value changes simultaneously. In contrast to the Cartesian system, the variable t actually changes. This representation allows a better viewing of the co-variational aspect (Goldenberg, Lewis, & O'Keefe, 1992).

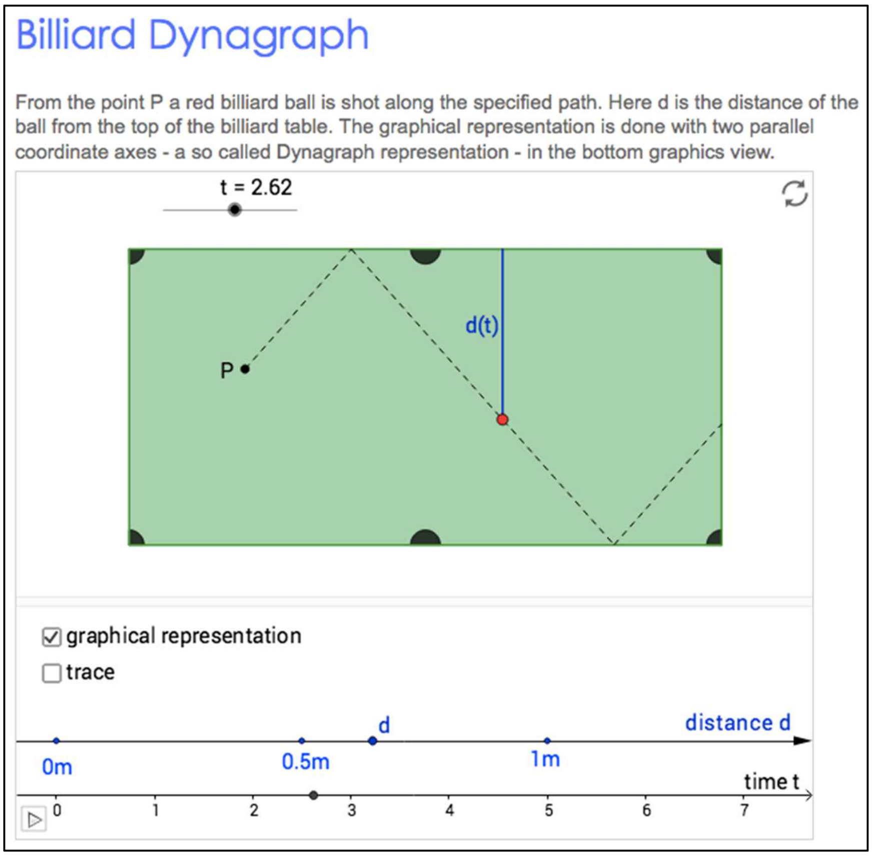

Fig. 1: Screenshot of GeoGebra applet "Billiard Dynagraph"

As the focus are students who have little experience in working graphs – mostly with path-time-diagrams – and no experience at all with the function concept the focus is the representational transfer between situational model and graph.

These problems and considerations lead to the following interesting aspects.

- What conceptions with particular attention to pre- or intuitive conceptions emerge concerning functional thinking of students of lower secondary schools?

A special interest is, if there are stages of intuitive conceptions that show a different level of conceptual understanding.

- How should dynamic materials be designed to support students of lower secondary schools to develop appropriate mathematical conceptions concerning functional thinking?
- What is the influence of dynamic materials on conceptions and internal representations of students of secondary schools particularly with regard to the understanding of the co-variational aspect of functional thinking?

2 Research design

The main focus of the study is the third aspect concerning the influence of dynamic materials on the students' conceptions. The questions are examined in more detail by means of a qualitative hypothesis-generating exploratory study. Therefore, the developed GeoGebra applets are used in a 7th grade of a lower secondary school in Austria. The qualitative analysis refers, among others, to methods of Grounded Theory (Strauss & Corbin, 1996).

The data collection is divided into a process that involves several stages. First 28 students, who had some experiences with graphs but none with the concept of function itself, participated a diagnostic test concerning different pre- and misconceptions. The test is based on different tasks from literature concerning conceptions (Schlöglhofer, 2000; De Bock et al., 2002) as well as a test instrument called CODI (Nitsch, 2015).

Based on the test results and answers to specific tasks some students were chosen for diagnostic interviews (Zazkis & Hazzan, 1999; Hunting, 1997) in order to obtain more information about the students' individual conceptions. After that, the students worked by themselves in pairs for a period of three lessons with several dynamic GeoGebra applets guided by accompanying tasks. 10 students were recorded during this period by audio-, video- and screenrecordings.

After completing this stage another diagnostic test with slightly varied tasks and several diagnostic interviews were carried out based on the test answers and the recordings.

A detailed qualitative analysis of the observational data and the interview recordings should give an insight into the conceptions of the students concerning functional thinking as well as the influence of the dynamic worksheets on these. The aim is to formulate hypotheses on the research questions.

3 First results

Here is one task of the pilot version of the first diagnostic test according to an example from Schlöglhofer (2000). The picture shows a trapezoid. The students had to choose one diagram out of three, which shows the grey marked area left of the dotted line as a function of the distance x, and to reason their decision.

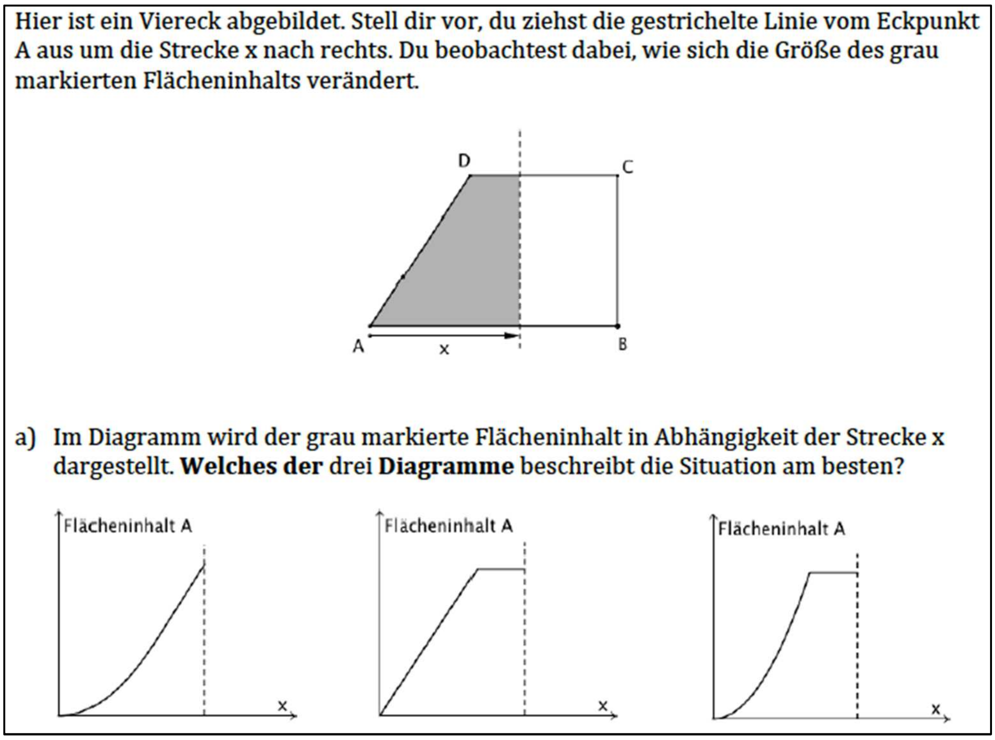

Fig. 2: Screenshot Task "Area" from the pilot test according to Schlöglhofer (2000)

The answers can be categorized based on the level of understanding and reasoning and show different levels of conceptions.

Category	Example of students answer
Some students made a graph-as-picture error and reasoned their answer just with the shape of the graph.	Because the drawing at the top looks exactly like the one I have circled.
Some students already argued with the area but they still took the shape into account.	Because the area increases and then stays the same.
Some students chose the correct answer with an explanation as quoted or similar.	Because the grey piece is getting bigger …

Table 1: Categories of answers to the task “Area”

There were also students who chose the correct diagram but the following reasoning showed that the students did not interpret the diagram correctly. A more detailed analysis will give further insight and will show the influence of the applets on the students’ conceptions.

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