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Developing Mathematical Language Proficiency in Preservice Teacher Education: A Case Study

TSG 31: Language and communication in mathematics education

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Abstract

The paper presents a method to support the development of mathematical language proficiency in preservice teacher mathematics education, including the oral component. In a case study a written imaginary dialogue task concerning algebraic word problems was used to enter into a dialogue with preservice teachers about their mathematical language. The aim was to raise the preservice teachers' awareness of their mathematical speech and point out possible consequences for the mathematical learning of their later students.

Mathematische Sprachkenntnisse in der Lehramtsausbildung entwickeln: eine Fallstudie

Zusammenfassung

Der Artikel stellt eine Methode vor, mit der die Entwicklung mathematischer Sprachkenntnisse, einschließlich der mündlichen, in der Lehramtsausbildung unterstützt werden kann. In einer Fallstudie wurde eine Aufgabenstellung zum Thema „Textaufgaben in der Algebra" in Form eines erdachten Dialoges eingesetzt, um mit den Lehramtsstudierenden in einen Dialog über ihre mathematische Sprache zu treten. Das Ziel war, die Aufmerksamkeit der Lehramtsstudierenden auf ihr mathematisches Sprechen zu lenken und mögliche Folgen für das Lernen ihrer späteren Schülerinnen und Schüler aufzuzeigen.

Keywords:	*Schlüsselwörter:*
Educational research	Bildungsforschung
Mathematical language proficiency	Mathematische Sprachkenntnisse
Preservice Teacher Education	Lehramtsausbildung
Mathematical writing	Schreiben im Mathematikunterricht
Imaginary dialogues	Erdachte Dialoge

1 Introduction

For participation in mathematical practice, the development of linguistic competencies and knowledge is essential (cf. Morgan, Craig, Schuette, & Wagner, 2014). Similarly, Prediger, Wilhelm, Büchter, Gürsoy, and Benholz (2015) state that "*language proficiency is the background factor with the strongest connection to mathematics achievement, among all social and linguistic background factors*" (p. 77). Clearly, a precondition for adequately supporting the students' mathematical-linguistic learning is a profound mathematical language proficiency of the teacher. Furthermore, not only the written mathematical discourse is relevant, but "*the spoken discourse of mathematics classrooms also has specialised features*" (Morgan et al., 2014, p. 845).

To support the development of mathematical language proficiency in preservice teacher mathematics education, including the oral component, the use of written *imaginary dialogues* (Wille, 2008; 2009; 2011) was

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studied on a course regarding secondary level mathematics education at the Alpen-Adria-Universität Klagenfurt, Austria, in 2015. A particular focus was on the preservice teachers' use of language concerning variables in algebraic word problems.

The aim of the paper is to present a method which has the potential to individually support the development of preservice teachers' specialised language of classroom dialogue, in particular language close to mathematical speech.

2 Theoretical framework

2.1 Algebraic word problems

Regarding algebraic word problems several learning difficulties can occur in the translation process from text to formula and the other way around. The use of variables in the mathematical discourse, one of the central concepts in algebra (cf. Schoenfeld & Arcavi, 1988), gives some indications of possible misconceptions that can lead to incorrect formulas or interpretations of terms. Seeing the development of mathematical knowledge as a change of the mathematical discourse (cf. Sfard, 2008, p. 255), it becomes apparent that inaccurate or vague use of mathematical concepts in the language points to possible learning difficulties. One example is the "*reversal error*" (Rosnick & Clement, 1980; Clement, Lochhead, & Monk, 1981; Wollman, 1983; Malle, 1993; González-Calero, Arnau, & Laserna-Beleguer, 2015) that can occur in a translation process from a multiplicative or additive relation of quantities to a formula[1]. Different causes entail the reversal error as *word order matching* and *static comparison* (González-Calero et al., 2015, p. 134). Whereas word order matching means a linear syntactic translation of the statement into the formula, static comparison is caused by a situation model built by the student where variables do not represent quantities but labels or units. Therefore, a student's or preservice teacher's use of variables as labels or units in the language can indicate static comparison and thereby cause the reversal error.

2.2 Imaginary dialogues between mathematical orality and literacy

To study how preservice teachers speak algebraically interviews or videotaped group works are conceivable, but this requires a considerable outlay. Using *imaginary dialogues* is a different approach in order to react *within the same course* for the purpose of entering in a dialogue with the participants. An imaginary dialogue is a form of mathematical writing where *a single student* composes a written dialogue between two protagonists who discuss a mathematical task or question (Wille, 2008). When finished, the imaginary dialogue is similar in appearance to a written dialogue within a theater script.

Mathematical writings have been investigated in different forms and terms (cf. e.g. Burton, 1985; Borasi & Rose, 1989; Clarke, Waywood, & Stephens, 1993; Shield & Galbraith, 1998). One characteristic of imaginary dialogues is that they possess a *middle position between mathematical orality and literacy*, because the conceived *spoken* utterances of the protagonists compose one written utterance[2] of the writing student (cf. Wille, 2017). Thus, the conceived spoken utterances have properties of the *language of immediacy* (cf. Koch & Österreicher, 1985), whereas the written imaginary dialogue as a whole utterance displays characteristics of the *language of distance*. Thereby, the mathematical discourse within an imaginary dialogue is *written speech*, but at the same time *conceived oral speech*.

In the studies of the author, it turned out to be useful to formulate an *initial dialogue* that contains the mathematical task or question and give it to the writing student in order to be continued.

3 Method

The study was carried out in two parallel courses with altogether 48 preservice mathematics secondary teachers at the Alpen-Adria-Universität in Klagenfurt, Austria, in 2015. The subject of the course was lower secondary level mathematics education including among other topics elementary algebra. Learning difficulties of students concerning word problems in elementary algebra were discussed, including the *reversal error* and its possible causes.

In a homework (done by 31 participants) the preservice teachers had to choose one specific learning difficulty or error that might occur in a school class. Next, they formulated as a single work an initial dialogue in order to develop an imaginary dialogue task for the elementary algebra students concerning the chosen learning difficulty. After that, the initial dialogues of the preservice teachers were commented on by the author and good examples were discussed in a following group work.

The focus of this study is the mathematical-linguistic proficiency of the preservice teachers that manifests itself within their initial dialogues with the underlying hypothesis: *Because of the closeness to spoken language the use of imaginary dialogues has the potential to reveal mathematical oral language inaccuracies of preservice teachers and therefore give the opportunity to address these. Thereby, the mathematical-linguistic proficiency of preservice teachers can be developed further.*

The initial dialogues were originally written in German and translated by the author.

4 Findings

In about a quarter of the initial dialogues of the preservice teachers variables were used as labels or units. For example the following sentences occurred:

"Is the x the girls or the boys?"

"Fire department X has 3 times as many fire engines as fire department Y. (...) Must X not be bigger, because X has 3 times as many cars?"

"If one denotes my father with X and my mother with Y (...)"

Or writings, where it remains unclear if the variable is meant as the object itself or its quantity:

"C ... cookies; F... friends"

In one commendable case the author used an incorrect formula on purpose to discuss why it is not correct. Interestingly, in all of the other initial dialogues where variables were used as labels or units only correct formulas were written down. Thus, in these cases the inaccuracies seem to be abbreviations and *not* an indicator for a situation model built by the preservice teachers where variables represent labels or units. Hence, in formal mathematical tasks it is possible that these preservice teachers would have written everything correctly. In comparison, by means of the initial dialogue task in the course, it was possible to address the inaccuracies in the language that could cause problems for the students' learning later in class.

A different group of preservice teachers (slightly overlapping with the preceding) constructed tasks where the use of variables is not meaningful or completely meaningless. For example, in one initial dialogue the main question was, why 1kg of cotton wool and 1kg of sand "*are the same*". Here, their language use revealed misconceptions of the mathematical notion of variable on the one hand and an unawareness of how variables can be used meaningfully in mathematics on the other. Within the course, again, it was possible to discuss the matter.

5 Summary

In about 40% of the initial dialogues, inaccuracies in the language concerning variables or misconceptions became apparent and therefore it was possible to raise the preservice teachers' awareness for these matters in the course. This holds a certain potential for the further development of their mathematical language proficiency regarding algebraic word problems. The middle position between mathematical speech and writings of imaginary dialogues seemed to be the key factor of this study. Of course, further studies in different mathematical areas and with modified writing tasks for the preservice teachers are necessary to confirm the argumentation for general cases.

References

Bakhtin, M. M. (1986). *Speech genres and other late essays* (V. W. McGee, Trans.). University of Texas Press Austin.

Borasi, R. & Rose B. J. (1989). Journal writing and mathematics instruction. *Educational Studies in Mathematics*, 20(4), 347-365.

Burton, G. M. (1985). Writing as a Way of Knowing in a Mathematics Educational Class. *Arithmetic Teacher*, 33(4), 40-45.

Clarke, D. J., Waywood, A., & Stephens, M. (1993). Probing the structure of mathematical writing. *Educational Studies in Mathematics*, 25(3), 235-250.

Clement J., Lochhead, J., & Monk, G. S. (1981). Translation Difficulties in Learning Mathematics. *American Mathematical Monthly*, 88(4), 286-290.

González-Calero, J., Arnau D., & Laserna-Belenguer B. (2015). Influence of additive and multiplicative structure and direction of comparison on the reversal error. *Educational Studies in Mathematics*, 89, 133-147.

Koch, P. & Österreicher W. (1985). Sprache der Nähe – Sprache der Distanz. Mündlichkeit und Schriftlichkeit im Spannungsfeld von Sprachtheorie und Sprachgeschichte. In O. Deutschmann et al. (Eds.) *Romanistisches Jahrbuch*, vol. 36 (pp. 15-43). Walter de Gruyter Berlin/New York.

Malle, G. (1993). *Didaktische Probleme der elementaren Algebra*. Vieweg Braunschweig, Wiesbaden.

Morgan, C., Craig, T., Schuette, M., & Wagner, D. (2014). Language and communication in mathematics education: an overview of research in the field. *ZDM – The International Journal of Mathematics Education*, 46, 843-853.

Prediger, S., Wilhelm N., Büchter A., Gürsoy E., & Benholz, C. (2015). Sprachkompetenz und Mathematikleistung – Empirische Untersuchung sprachlich bedingter Hürden in den Zentralen Prüfungen 10. *Journal für Mathematik-Didaktik*, 36, 77-104.

Rosnick, P. & Clement, J. (1980). Learning without understanding: the effect of tutoring strategies on algebra misconceptions. *Journal of Mathematical Behaviour*, 3(1), 3-27.

Schoenfeld, A. H. & Arcavi, A. (1988). On the meaning of variable. *Mathematics Teacher*, 81(6), 420-427.

Sfard, A. (2008). *Thinking as Communicating. Human Development, the Growth of Discourses, and Mathematizing*. Cambridge University Press Cambridge.

Shield M. & Galbraith, P. (1998). The analysis of student expository writing in mathematics. *Educational Studies in Mathematics*, 36(1), 29-52.

Wille, A. M. (2008). Aspects of the concept of variable in imaginary dialogues written by students. In O. Figueras, J. Cortina, S. Alatorre, T. Rojano, & A. Sepúlveda (Eds.), *Proc. 32nd Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 4, pp. 417-424). PME Cinvestav-UMSNH, Mexico.

Wille, A. M. (2010). Steps towards a structural conception of the notion of variable. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proc. of CERME 6*, Jan. 28th-February 1st 2009, Lyon, France (pp. 659-668). http://www.inrp.fr/editions/cerme6. INRP France.

Wille, A. M. (2011). Activation of inner mathematical discourses of students about fractions with the help of imaginary dialogues: a case study. In B. Ubuz (Ed.), *Proc. 35th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 4, pp. 337-344). PME Ankara, Turkey.

Wille, A. M. (2017). Imaginary Dialogues in Mathematics Education. *Journal für Mathematik-Didaktik*, 38(1), 29-55. doi: 10.1007/s13138-016-0111-7.

Wollmann, W. (1983). Determining the Sources of Error in a Translation from Sentence to Equation. *Journal for Research in Mathematics Education*, 14(3), 169-181.

[1] Rosnick and Clement (1980) describe the reversal error regarding the following task: “Write an equation using the variables S and P to represent the following statement: ‘There are six times as many students as professors at this university.’ Use S for the number of students and P for the number of professors” (p. 4).

[2] In this paper the notion *utterance* is understood as Bakhtin (1986) describes it as determined by its beginning and end which are determined by the change of the writing or speaking person (cf. p. 71).