Comprehension of calculus concepts based on motion sensor data

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1. Introduction

Results from physics education research indicate that laboratory activities using a motion sensor can supply a powerful setting for addressing students' difficulties in interpreting graphs of kinematics variables (Douglas et al., 2000; Zucker et al., 2012). Furthermore, real-time graphing of data appears to be a key feature for both cognition and motivation as it allows students to process information about a physical event and its graph simultaneously (Brasell, 1987). In particular, students bring a unique level of understanding of the data to the graph when the data comes from an experiment towards which students feel a sense of ownership (Mokros & Tinker, 1987). In addtion, graphs are also fundamental mathematics tools, whose application requires various types of mathematical competencies (Uzun et al., 2012).

According to early research, student difficulties appear in understanding the function concept in interpreting graphs, particularly, those graphs in which a variable is time-dependent, as for example distance-time or velocity time graphs (Clement, 1989). Many students at high school, or even university level, lack the ability to understand and interpret graphs in physics as student knowledge emerges to be very compartmentalized (Planinic & Milin-Sipus, 2012) in two distinct contexts; the physics world and the mathematical world. Although mathematics, as the language of data, is viewed as an important tool in physics classes, students demonstrate a great resistance to apply their mathematical knowledge to physics. The mathematical world is full of rules which relate to x and y, and to coefficients such as a, b and c (Woolnough, 2000). In consequence, there are many graphs which are characterised by x and y axes, mostly with scales that are without units. The physics world concentrates on specific rules accompanied by a variety of equations and experiments, often ending up with graphs which describe what happened in the experiment. Although it would be worthwhile to make substantial links between mathematics and physics, physics is not generally considered as a tool for teaching mathematics. However, results from own studies suggest that incorporating real-world data-collection tools into the mathematics classroom can add a new dimension to the teaching of mathematics concepts (Urban-Woldron, 2007).

The focus of the paper is on activities to develop conceptual understanding of fundamental calculus concepts, specifically instantaneous rate of change and accumulation of area under a curve by introducing motion sensor devices to the mathematics classroom. The activities are based on the use of a motion detector in combination with a TI-Nspire CX CASTM calculator. Beyond enhancing the teaching of mathematics by making it more realistic, alive and accessible, the motion sensor device enables users to get real time x(t) and v(t) graphs of a moving object or person. Consecutively, these graphs can be productively used to introduce differentiation and integration on visual levels. Apart from viewing functions symbolically, graphically, numerically, and verbally, the linking-up with other disciplines and real applications can be a further valuable attempt for enhancing the teaching of mathematics by integrating technology extensively and even more versatile into the mathematics classroom. In this context, the author presents examples based on real experiments where functions can be viewed from their characteristic behaviour-over-time/rate-of-change patterns, explaining and discussing the steps of such an approach in further detail.

Using the motion detector as early experiences for students at each level allows repeated reference to the motions and their interpretations on the graph. Therefore, a starting point with steady walks in front of a motion detector can centre a discussion on the characteristics of the motion that caused the graph to be linear and that the slope is dependent upon the speed. Consequently, this connection between speed and slope is used as the foundation concept for the qualitative and quantitative study of all other functions. Next, the exercises which start with motion producing straight line graphs evolve to demonstrate first and second derivative concepts as well as the integration procedure. Attendant, some of the vocabulary is introduced in the motion detector activities by using "characteristic behaviour-over-time" in addition to "rate of change" to describe the standard linear, quadratic, exponential, and periodic functions.

In fact, the technology use in the classroom challenges teachers to re-examine teaching approaches in order to fully exploit the potential of the resources. Calculus, as the mathematics of change and accumulation of quantity, is conceived as a context in which the use of data from real experiments can significantly contribute to conceptual understanding of mathematics concepts. For example, Metcalf and Tinker highlight the general beneficial affordances of technological learning environments: *"Technology is needed…not just to give students exposure to the technology or to satisfy parents; technology greatly improves learning and supports science education standards that are difficult to teach without using technology"* (Metcalf & Tinker, 2003). According to concepts of graphs in kinematics, it is generally agreed that an important component of understanding the connection between reality and the relevant graphs is the ability to translate back and forth in both directions (Mc Dermott et al., 1987). Rosenquist and Mc Dermott (1987) suggest an early approach in which actual motions in the laboratory provide the basis for developing a qualitative understanding of the kinematical concepts. They emphasize how instruction based on the direct observation of motion can help students recognize key features of definitions and make explicit connections among concepts and their graphical representations.

Furthermore, Thornton and Sokoloff (1990) argue that the Microcomputer-based Laboratory (MBL) tools give students the opportunity to experience the excitement of the process of science. Students can creatively build and test models and explain the world around them. *"Because of their ease of use and pedagogical effectiveness, they make an understanding of physical phenomena more accessible to the naïve science learner and expand the investigations that more advanced students can undertake"*. However, the authors underline that the tools are far from sufficient; there is need for a combination of tools and appropriate curricular materials as well as eligible social and physical settings. Research indicates that it is the real-time nature of MBL that accounts for the improvement in student achievement (Brasell, 1987). Students are able to examine the situation while the graphs relating to the specific event are being produced (Beichner, 1990). Five features of MBL seem to contribute to its success in facilitating graphical communication: MBL pairs, in real time, events with their symbolical graphical representations; it provides immediate feedback; it eliminates the drudgery of graph production; it encourages collaboration, and it affords genuine scientific experiences. Thus, students are able to concentrate more on discovering and understanding physics concepts, and in consequence, develop critical thinking like a scientist.

In addition, advances in technology have introduced remote data logging, calculator-based laboratories (CBL) and hand-held devices over the past years. A Calculator-Based Ranger (CBR), a motion detecting device, enables real data from real experiments to be captured in real time. In turn, using a CBR with graphing calculators permits rapid production of visual displays, with statistical data available for analyses leading to prediction models. Furthermore, these experiences help students see the groundwork of existing interrelationships between underlying concepts. For example, by using a CBR and a graphing calculator, students are expected to collect, view, and analyze motion data without tedious measurements and manual plotting. As a person walks in front of a motion detector back or forth, the motion of the person is plotted on the screen.

2. Using the CBR as an interactive and cognitive tool

The motion detector is essentially a sonar unit that sends out short pulses of high frequency sound, then detects and amplifies the echo. By the means of appropriate software, in connection with a calculator or a computer, the time between the transmitted and received pulse can be measured, and subsequently, position, velocity and acceleration of the object causing the reflection can be calculated and displayed. Whereas distance is the linear extent of space between two things, velocity is the rate of time change (speed) of an object, but in a specified direction. Furthermore, acceleration is the rate of time change of velocity with respect to magnitude or direction. One of the most exciting features of the motion detector is its ability to detect and immediately display graphs of the motion of any object (Thornton & Sokoloff, 1990).

A review of studies addressing the effect of using a motion sensor highlights two important points: (a) the use of a motion sensor is motivating and satisfying for students at a variety of educational levels, and (b) the effectiveness of using a motion sensor is greatly enhanced by appropriate instruction (Nakhleh, 1994). Physical experiments have a remarkable potential in mathematics lessons. Derived from case studies, it seems possible even for nine-year olds to interpret graphs generated through interactions with sensors (Mokros & Tinker, 1997). Eleven-year old students can gain an intuitive understanding of basic calculus concepts by using a position sensor with a computer that generates a real-time graph of the student's motion and velocity (Urban-Woldron, 2010). Gathering data in a hands-on and real time method helps classrooms coming alive and promotes the development of understanding the connections between mathematics and physics (Urban-Woldron, 2006). Findings from studies indicate that even single-class-period activities using a motion sensor improved students' ability for interpreting graphs (Brasell, 1987). Real-time experiments with the motion sensor allow students to "see" and sometimes also to "feel" the connection between a physical event and its graphical representation (Beichner, 1990).

The CBR has a built-in microprocessor that does much more than measuring the time interval between transmitting an ultrasonic impulse and the first returned echo. When the data is collected, the CBR calculates the distance of the object from the CBR using a speed-of-sound calculation. Then, the first and second derivatives of the distance data with respect to time are computed. By this way, velocity and acceleration data are obtained and stored in particular variables. Therefore, an interesting mathematics classroom activity could be to perform the same calculations as the CBR by hand. Specifically, the velocity of the object can be calculated by using the sample times in the variable "Time" and the distance data stored in the variable "Position". Finally, the calculated results can be compared to the velocity data provided in the variable "Velocity" by the CBR.

$$Velocity_n = \frac{(Position_{n+1} + Position_n)/2 - (Position_n + Position_{n-1})/2}{Time_{n+1} + Time_n}$$

Similarly, the acceleration of the object can be calculated by hand using velocity data from the CBR or the student-calculated values in conjunction with the samples times provided in the variable "Time".

2.1 Linear functions: Walking in front of a motion detector

The motion detector, combined with appropriate software, gives the students power to explore, measure, and learn from the physical world (Thornton, 1985) and encourages them to build effective links between mathematical functions described by x and y and the real world of moving objects. From a mathematical point of view, the students are enabled to make links by using data from real experiments and viewing them in different representations: tables, graphs, functions and equations. Moreover, the students can grasp the concept of slope as a rate of change and find out that the area under a velocity-time graph within a particular time interval represents the distance travelled by the object. The collection of data in the real experiments does not need much time; for each single event only a few seconds. Most of the time can be spent on reasoning, documenting predictions, observations and results and explaining the whole activity in own words highlighting what was learned. Immediately after walking with roughly constant speed in front of the motion detector, the plotted graph can be traced in order to combine real motion and graphical representation and make students familiar with visualization of motion (see fig. 1). Students should detect linear parts in the graph and cognitively connect them to the real motion to get an idea how velocity is expressed in a position-time graph.

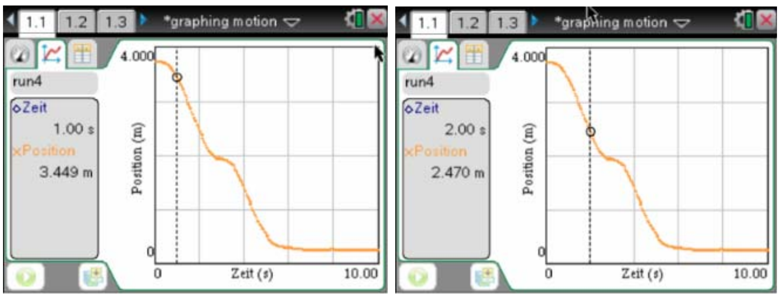

Figure 1. Position vs. time graph (t = 1s and t = 2s)

Motion sensors used along with graphing devices allow graphs to be viewed while the data are collected. As a student walks towards or away from a motion sensor, the position versus time graph of his/her walking is immediately visualized. It is possible to determine in what direction an object is going, how fast it is moving, how far it travelled and whether it is speeding up or slowing down. Particularly, students can find links between the graphical representation of the data and the physical movement of human bodies. By using particular data points students can determine the average speed of the body motion which is roughly 1m/s for the motion displayed in figure 1. As the distance at t = 2s is smaller than at t = 1s, the person is moving toward the motion detector. During the time interval [6s; 10s] the person is not moving.

A special feature of TI-Nspire software, the activity "Distance vs. Time Graphing", supports the generation of random target distance graphs consisting of three linear parts (see fig. 2). Students can be asked to match a graph on the screen. When students are expected to walk toward or away from a motion detector to match a given graph they first have to interpret the shape of the graph. Therefore, the graph-match-activity addresses various student competencies: First, the students previously have to study the graphs and consider how they would walk to produce the target graph shown on the screen. Then they can test their prediction, choose a starting position and finally walk in the considered way to match the target graph on the calculator screen and get immediate feedback. Finally, when they are not successful the first time, the process can be repeated until the motion closely matches the graph on the screen.

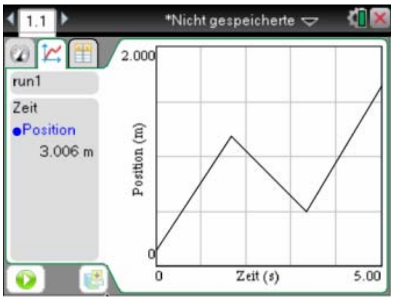

Figure 2. Position vs. time graph

In particular, students comprehend the meaning of slope by working out such tasks. In order to be successful in creating the graph shown in figure 2, they first have to figure out that the graph is made up of three linear pieces related to three time intervals. Next, they have to find out what distance they have to complete in each time interval. Thus, they learn to see how speed and velocity are related to the graph on the screen and how they have to operate their movement. Finally, they have to decide where to start. As they move, they get instantaneous feedback and realise how the movement of their own bodies is in connection with the displayed graph. They may need to try their walk several times until they get it close to the instant graph.

2.1 Quadratic functions: Bouncing balls and rebounding trolleys

The position of a bouncing ball which is released below a motion detector and is reflected on the floor is continuously measured and the data collected are analyzed (see left picture in fig. 3). In order to make the (time, position) graph more similar to the real movement of the ball, the measured data are transformed by subtracting them from the distance between motion detector and floor (see right picture in fig. 3).

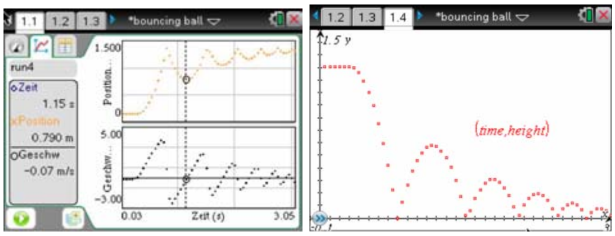

Figure 3. Motion of a bouncing ball (measured and transformed data)

Students are asked to describe the height of the ball as a function of time. Figure 3 suggests that each bounce can be described by a quadratic function $f(x) = a \cdot (x - b)^2 + c$, where b and c represent the coordinates of the vertex of each parabola. Related to the real experiment, c indicates the height at the point b of time. Students have to establish and solve systems of equations derived from the data in the graph for the coordinates of the vertex and one or two more points or use calculator-built-in regression functions. Thereby, they identify and derive a common value for the parameter '$a = -4.83$' for each parabola in accordance to the gravity law (see fig. 4). Furthermore, they learn how the functions, related to the first and the second parabola, are similar and how are they different. Finally, they get an idea how the parameters in the mathematical equations are connected to the real movement of the ball.

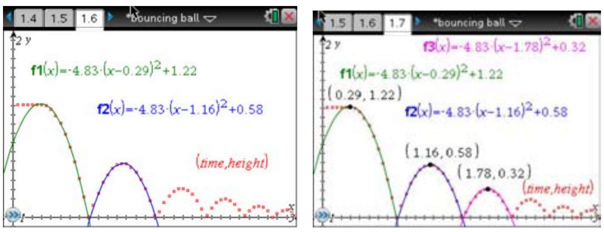

Figure 4. Adjusting functions to real data

In addition, further questions can be asked in the classroom. For example, addressing the speed of the ball by asking when it is highest and when does it occur or how does the velocity change as the ball rises and falls. Next, students could be asked to figure out the total distance travelled by the ball during the first five bounces in at least two different ways. Students can either calculate the total distance travelled by the ball when calculating the area under the curve in the velocity-time graph or sum up the y-coordinates of the maxima of the parabola, starting with the second, then multiply this sum by two and finally add the first one. From the graphs and also from the corresponding tables, students derive that the velocity constantly increases when the ball falls and decreases during raise. When students try out the experiment with different balls and distinct floors, they realize that the material of both has an influence on the shape of the graphs.

Next, students could be asked to find a model to describe the height of the ball for a particular bounce. By developing a table with the coordinates of the vertexes of the parabola and searching for an appropriate regression function, they discover that the maximum height of the ball decreases exponentially from bounce to bounce for each ball and its initial height. For $y = h \cdot p^x$, y is the current height, h is the initial height, p is a constant depending on the properties of the ball and the floor and x is the number of the bounce.

Figure 5. Experimental setup "Rebounding Trolley"

Looking more closely at graphs which "look quadratic", the activity, called "Rebounding Trolley" could help to emphasize the idea that velocity and acceleration can be in opposite directions. The data-logging experiment uses a motion sensor in combination with a TI-Nspire handheld device to record and analyze the motion of a trolley rolling down a slope and rebounding due to the action of a spring buffer (see gig. 5). The data-logging software of the TI-Nspire is configured to measure the distance of the trolley from the sensor and to present the results as a graph of position against time. After the trolley is released, it bounces off a wooden block and moves up and down the runway several times. Thereby, the distance travelled by the trolley reduces after successive bounces. Data are collected within a few seconds. Students obtain a graph of position against time as shown in figure 6 and have to associate the features of the graph with the observed motion.

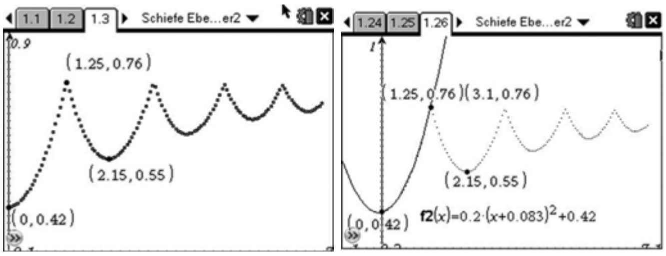

Figure 6. Position vs. time graph of the rebounding trolley

Students can trace the graph and identify interesting points on the graph. Finally, they could study the velocity of the trolley more directly by setting up the data collection software on the TI-Nspire to plot velocity against time. In contrast, they could be inspired to find ways to derive the velocity of the trolley from the position-versus-time graph. If they do so, they possibly end up discovering another interesting feature of the graph: the asymmetry of each loop.

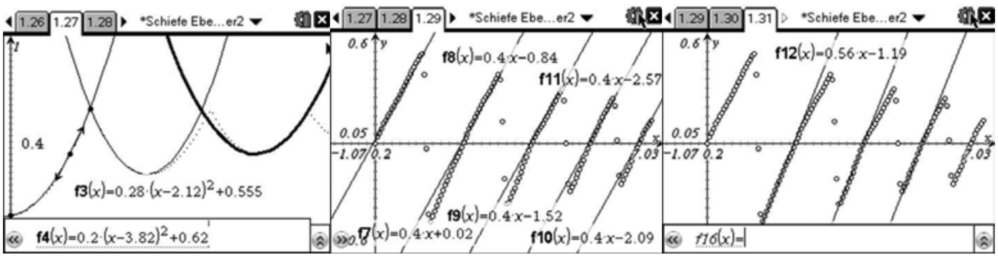

Figure 7. Parabolas do not fit the half loops- different velocities for downward and upward motion

The TI-Nspire offers students options to adjust polynomial functions to the collected data (see Fig. 7). The slope represents the instantaneous velocity of the trolley and can be easily explored by a point on the function. When students try to find out further polynomial functions fitting the other parts of the graph, they may find out probably the most interesting feature of the graphs: each half-loop is asymmetrical due to the presence of friction.

3. Conclusions and implications

Summarizing the activities, data obtained from real experiments and simple teacher questions can stimulate a rich discourse in the classroom and further exploration. Compared to the traditional instruments used in the classroom in mathematics teaching, more data can be very precisely acquired by using a motion detector in combination with TI-Nspire software in order to obtain the shape and equations of the corresponding curves. Consequently, students are encouraged to construct mathematical ideas by using educational technology as cognitive tools. Therefore, motion-detector tasks can stimulate cognitive activation of students in the mathematics classroom and can support and extend mathematical reasoning and sense making. In conclusion, implementing this kind of technology in the mathematics classroom can lead to a substantial change in the learning process.

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