



# TIME 2014

Technology and its Integration in Mathematics Education  
International Conference

Book of Abstracts

# Keynotes

## **Peter Baumgartner, Danube University Krems, Austria**

*The Potential of the Internet for Mathematic Education MicroLearning, MOOC, OER, ePortfolio and other virtual Monsters*

The Internet as a global system of interconnected computer networks can be used as a new kind of learning environment. The majority of its educational and didactical potentials are hidden and not exploited when it is simply regarded as presentation or delivering tool for subject material. In my talk I will critically review some educational trends as well as their application in Math Education.

[Baumgartner.pdf](#)

## **Bruno Buchberger, Johannes Kepler University, Austria**

*The Future of Mathematics: A Personal View and Comments on Math Education*

In 21st century, the dynamics of mathematics gains speed, energy, and sophistication by the application of mathematics to itself: The invention and verification process of mathematics itself can be supported by mathematical algorithms. In the talk, we will clarify the creative mechanics of self-application of mathematics and derive some consequences for mathematical research, education, and application.

[Buchberger.pdf](#)

## **Regina Bruder, Technische Universität Darmstadt, Germany**

*Technologiegestützt intelligentes Wissen und Handlungskompetenzen fördern*

Ausgehend von einem Überblick über die inzwischen in Studien und mehrjährigen Modellversuchen vielfach bestätigten Potenziale digitaler Werkzeuge für Mathematikverstehen und Anwendungen von Mathematik werden zunächst „Risiken und Nebenwirkungen“ bei einem konzeptionell einseitigen Technologieeinsatz mit Fokussierung auf prüfungsrelevante Aufgabenformate thematisiert. Der Mathematikunterricht wird nicht allein durch die Existenz von Technologie und zentrale Prüfungen „besser“! Es gibt jedoch erprobte Konzepte für einen gewinnbringenden Technologieeinsatz für die Lernenden, deren Kernideen vorgestellt werden. Darüber hinaus bieten neuere technologische Möglichkeiten wie Videotutorials und online-Diagnosetools Unterstützung z.B. beim individuellen Schließen von Grundlagenlücken und zur Selbsteinschätzung der Lernenden, wenn bestimmte Qualitätsstandards erfüllt werden.

[Bruder.pdf](#)

## **Pavel Pech, University of Southern Bohemia, Czech Republic**

### *The Use of DGS and CAS in Proving Theorems*

The use of dynamic geometry systems (DGS) and computer algebra systems (CAS) changed teaching geometry at all school levels considerably. To solve a problem students first visualize it by DGS then by changing parameters the problem is interactively modified and geometry properties like invariant points, lines, circles etc. are recognized. Using these knowledge a conjecture is stated and classically proved or disproved. But sometimes we do not have a *key idea* to find a classical proof. Then the use of CAS can help. By the theory of automated geometry theorem proving we are able to prove many such theorems. It turns out that integration of DGS and CAS is useful and helps to solve problems. This approach is demonstrated in a few examples of elementary geometry in a plane and space.

[Pech.pdf](#)

## **Gilles Picard, ÉTS Montréal, Canada**

### *Technology in teaching Mathematics: Looking Back and Looking Forward*

The students we now have in 2014 were almost born with technology in their hands. Numerical and graphical calculators are accepted tools on every student's desk. But they also have smartphones, tablets, ipads, with even more powerful apps, always at their sides. How does this change math education? What can we still expect them to be able to do "by hand" and where will they need technology to support them? Have we gained something while encouraging them to use all these tools?

We will show examples of math calculations that used to be done manually 30 or 40 years ago and that are accepted today as being done via technology. Today's students will smile if we attempt to explain that these calculations used to require a lot of time and paper. Looking at the way technology has evolved in the last 15 years, we believe we should be asking ourselves which math operations will make students smile in 30 years from now, thinking of how we did these calculations in 2014. Since numerical calculations are now mainly done with technology, we need to consider which level of algebra students really need to acquire, since calculators, apps and software can do more and more of these operations for them. Computer Algebra Systems (CAS) are easily available. Shouldn't we be showing them how to better use this technology? To be aware of its limits and caveats. Examples will be presented to illustrate this.

At our university, all engineering students are required to have a CAS calculator. This has been mandatory now for 15 years. We will discuss how this has changed the way we teach some of our topics and show examples of how technology has allowed us to continue to cover the same general curriculum, discarding some manual calculations and having students explore and work even more mathematics or more challenging problems. We will also present how students themselves evaluate this technology in their learning environment and how good they consider their teachers are in teaching and using this CAS environment in the classrooms.

[Picard.pdf](#)

## **Marlene Torres-Skoumal, IB Examination Board**

### *The International Baccalaureate External Examination Model*

Curriculum-based external exit examinations systems are world wide the norm rather than the exception. Proponents of this system argue that imposing a curriculum-based external examination system improves student achievement and university success. Recent international multivariable statistical studies have shown a significant difference in student school exit and subsequent success among nations who employ this system. The main reason for this success is that well-designed external exit examinations induce improvements in instructional practices. This keynote will address the traits and achievements of the IB system with the hope that it can aid in the discussion and design of Austria's external mathematics Matura.

[Torres\\_Skoumal.pdf](#)  
[Torres\\_Skoumal\\_appendix 1.pdf](#)  
[Torres\\_Skoumal\\_appendix 2.pdf](#)

# Long Lectures

## **Joseph Abi-Nader, Maplesoft**

*New Ways to Enhance the Classroom Using Symbolic Computation: Automated Assessment, Modeling and Simulation, and More*

From its beginnings as a software tool for math researchers, Maple has developed into an extremely powerful and easy-to use platform for technical education, suitable for teaching and learning math, physics, finance, engineering, and other technical subjects.

With Maple as the foundation, Maplesoft offers solutions for every aspect of technical education, including e-learning, automated testing and assessment, and modeling and simulation. In this presentation, you'll learn how Maplesoft innovations like Clickable Math, Maple T.A., and MapleSim are empowering educators, motivating students, and revolutionizing the classroom.

[Abi-Nader.pdf](#)

## **Michel Beaudin, ETS Montréal, Canada**

*Using TI-Nspire 2D Graphs in a CAS Environment*

There are up to seven 2D plot windows in Nspire CAS. In this talk, we won't give a detailed description of the different features of each; instead, we will start by plotting a simple curve - using many Graph Entry/Edit styles. This may seem easy but, for students using Nspire CAS technology in the classroom, this must be learned and understood.

Plotting these graphs will become an opportunity to use some nice features of Nspire CAS: namely the power of the math engine, the built-in geometry package and the possibility of using animations.

Keywords: 2D graphs, CAS, animation.

[Beaudin\\_2D.pdf](#)

[Beaudin\\_2D.tns](#)

## **Tim Bebensee, CASIO**

*CASIO Class Pad II - Praxis*

Der CAS-Rechner ClassPad II ist einerseits Nachfolger des ClassPad 330, der schon 2003 mit einer revolutionären Bedienung beeindruckte. Andererseits ermöglicht das 4,8 Zoll große, hochauflösende und berührungssensitive Farbdisplay eine Arbeitsweise wie bei einem Tablet-Computer. Im Gegensatz zu einem solchen, konzentriert sich der ClassPad II aber auf den Einsatz in den naturwissenschaftlichen Fächern. Ergänzt durch die schnelle Startzeit und die einfache Bedienung lässt sich das Gerät schnell gewinnbringend in den Unterricht integrieren. In diesem Vortrag wird die grundlegende Arbeitsweise des ClassPad II anhand einiger Beispiele demonstriert. In einem Praxisteil können verschiedene Aufgaben mit Hilfe des ClassPad II gelöst werden.

[Bebensee.pdf](#)

## **Josef Böhm, ACDCA & DUG, Austria**

### *Inspirations from Outside*

As a regular reader of journals, daily and weekly newspapers, digital newsletters, etc, I got many inspirations for treating the presented problems or ideas by means of a CAS. It is obvious that tables presented in scientific and economic journals inspire for graphic representation and statistic investigations. This will not be treated in this talk.

My examples shall demonstrate that one can find interesting and challenging problems for applying a CAS not only in textbooks. These problems can be for students of various mathematical levels and just for one's own entertainment. Among these examples are: modelling a baroque pillar with an interesting cross section, finding an analytic proof for a geometric problem, investigating the equation of a locus which was generated by a dynamic geometry program, modelling a special evolution strategy, and modelling the pricing of call & put options. Solving brain twisters discovered in journals or in respective collections of such problems with a CAS need other competencies than solving them by reasoning (see also my respective workshop).

A very special inspiration was given by working with LUA – a programming language which can be used with TI-NspireCAS. Clifford Pickover's "Mygalomorphs" get a new quality by applying sliders for changing shape and colours rapidly. This realisation of the "inspiration" was the result of a fruitful communication with a Swiss colleague.

In my demonstration I will use mainly DERIVE and TI-NspireCAS – because I am most familiar with these both systems.

[Boehm\\_Inspirations.pdf](#)

## **Andreas Daberkow, Oliver Klein, Heilbronn University, Germany**

### **York Xylander, bettermarks GmbH, Germany**

#### *Online Mathematics for Bachelor Students – it really works!*

Many bachelor students incomings at Technical Universities of Applied Sciences have severe gaps in fundamental mathematics. These gaps lead to a large drop out rate not only in university mathematics but also in further important technical subjects like engineering mechanics, physics or electrical engineering.

After a test trial over three terms Heilbronn University has established a sustainable practice and assessment process to adjust the math skills for incoming students. This process is based on an online mathematical learning and assessment tool which has been developed by teaching and didactics experts from the bettermarks company. Primarily designed for German school usage up to the 10th grade, the online learning system was adapted to the university needs and to the integration into the university learning management system ILIAS [1].

The presentation covers all approaches that have been made to implement the tool at Heilbronn university [2]. A reliable mathematical online tool with a rich content and with an attractive graphical user interface, an obligation of students to participate and a well-structured service and assessment process are the key factors for a successful implementation. In the winter term 2013/2014, more than 400 students from 9 different study courses participated in the online math training program. Over 60% of the students recommend the mathematical tool and the related process to other students.

After more than 2 years and 4 terms of experience the authors are convinced that this approach is promising to lower the drop out rate and also to inspire the students for mathematical content.

[Daberkow.pdf](#)

**Jean-Jacques Dahan, IREM of Toulouse, France**

*Techniques of Morphing to Stimulate the Teaching with Technology Examples within the TI-Nspire Environment and Cabri 3D*

“Continuity” is one of the two principles underlying dynamic geometry software, the other one is “determinism” (Bernard Geneves PhD 2004). We know that we cannot respect together both of these principles in a DGS. Nevertheless, developers of such software have always tried to respect these two principles in most of the cases that we can meet in working with such software. Everybody is now aware that continuity in dynamicity is the revolution in visualisation of mathematics (Dahan J.J., Modeling Cha Cha dance within the TI-Nspire and Cabri environments, Time 2008) and now it is time to show to the teachers the richness of this principle to model “morphing” (transforming continuously connex objects onto other ones) when using DGS. This paper presents within the TI-Nspire environment several techniques to morph for example:

1. some number onto another one, some polygons onto others or onto a circle
2. some curve of function onto another one, some parametric or polar curves onto other ones or onto polygons
3. some circle onto ellipses to model rotation in parallel perspective
4. some surface  $z_1 = f_1(x,y)$  or parametric surface onto another one in military perspective (Geometry application) or in central perspective (3D graphing tool)

Lots of other examples will be presented at all levels using the CAS part of the software such as the morphing of the intermediate Fourier sums onto the Fourier sum

or the morphing of a flag floating in the air

The secret of all the techniques used here is based on the proportionality and how to respect it in using dilation in geometry, and barycenter formulae in the CAS.

A survey of 3D examples under Cabri 3D will show the power of these techniques .

Finally, this paper aims to provide ideas to teachers for using technology to treat motivating problems needing necessarily technology (at all levels).

Dahan.pdf  
Dahan.zip

**Thierry Dana-Picard, Giora Mann, Nurit Zehavi, Israel**  
*The Study of Envelopes in a CAS Environment*

In considering future developments in Mathematics Education, a graduate course, *Integration of methods in mathematics through a Computer Algebra System (CAS)*, is offered to Israeli Mathematics teachers at the Weizmann Institute of Science. The course is intended to extend their mathematical knowledge and perspectives. For this purpose we present specific topics, rather than whole subjects, that can be studied through a variety of mathematical methods. (Mann and Zehavi, 1998; Zehavi, 2004).

It is our responsibility and challenge as educators to identify pedagogical advantages of the technology for broadening learning opportunities. One way to do so is by introducing topics from the classical mathematics as recommended by Cuoco and Levasseur (2003). Topics that were considered too technical for students become tractable by using the technology to perform the algebraic manipulations and to construct visualization. The illustration of classical concepts and methods in differential geometry using CAS technology provides examples of the new educational possibilities (Migliozzi 1995, Florence 1997).

However, envelopes are not just a beautiful classical topic in differential geometry; it can be studied with various methods from Algebra, Geometry and Calculus. Thom (1962) complained about its disappearance from the curriculum. We claim that the rich variety of methods invite the construction of mathematical relationships.

Using a computerized environment, especially a Computer Algebra System, we may have a revival of a classical topic.

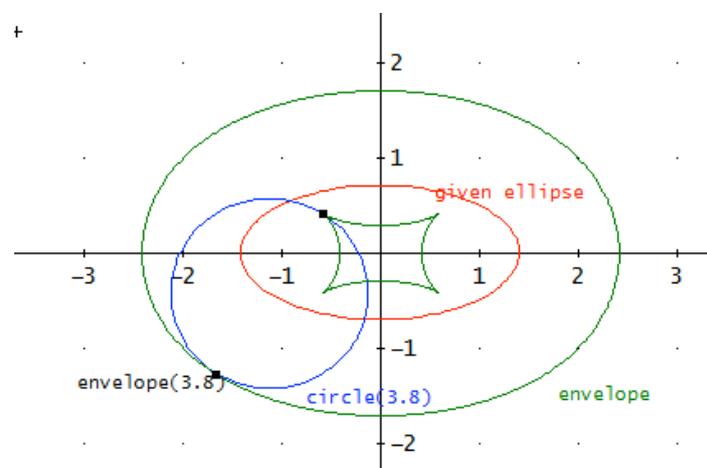
First we show a couple of classical examples, among them:

1. The family of tangents to a given plane curve.
2. The family of curvature circles of a given plane curve.

The CAS based tasks enable the student to develop the needed skills to deal with more complicated situations. In particular, the usage of animations and of slider bar implemented into Derive transforms the study of envelopes from a static study of curves to a dynamical one, by sliding curves and points to build the envelope.

The parallel usage of different registers of representations is multifold. On the one hand, we switch between implicit and parametric representations of curves. On the other hand, using technology, we switch between algebraic, numerical and graphical.

The dynamical features provided by Derive (or other software like Maple, GeoGebra, etc.) makes experimentation worth. The concept of an envelope is made concrete via its CAS realizations. We illustrate this in advanced situations. In particular, we focus on the appearance of envelopes with more than one component and also with singular points.



**Jon D. Davis, Western Michigan University, USA**

*Prospective Mathematics Teachers' Use of the Symbolic Manipulation Features of a CAS in their Lesson Planning*

Although secondary mathematics teachers in the U.S. have been hesitant to make use of the symbolic manipulation features of a computer algebra system (CAS-S) (Braswell et al., 2001) we may now be on the cusp of a shift in teachers' attitudes towards and consequently their use of this technology. Recently, three secondary mathematics textbooks have become available for U.S. teachers that incorporate CAS-S to varying degrees (Cuoco, 2007; Fey & Hirsch, 2007; Usiskin, 2007). This study had three goals. First, it used a research-based analytical framework (Davis, under review) to examine the use of CAS-S by curriculum developers in one lesson appearing within these three textbooks. Second, the framework was used to analyze the use of CAS-S by ten prospective secondary mathematics teachers (PST) in their lesson plans based upon these textbooks. Third, PST also justified CAS-S in their lesson plans and these explanations were categorized using a combination of open coding and in-vivo coding methods (Strauss & Corbin, 1998) and used as a window into their beliefs about how and when this technology should be used by students.

In the first lesson, textbook developers intended to provide students with the opportunities to solve equations involving quadratic functions using tables, graphs, and CAS-S. In the second lesson, textbook developers asked students to use CAS-S to explore the simplification of symbolic expressions. The majority of the PST followed the textbook developers' intended use of CAS-S. The textbook developers in the last lesson did not explicitly advocate CAS-S use as students learned how to find the magnitude and direction of complex numbers using paper-and-pencil procedures. In this lesson, the majority of PST were willing to explicitly ask students to use the technology. Across all three lessons PST were willing to incorporate equal to higher levels of CAS-S than the textbook authors. This suggests that the incorporation of this technology in secondary mathematics textbooks has the potential to increase the use of CAS-S by teachers in classrooms.

A theme appearing in one PST's (Chris) lesson plan was the importance of the strategic use of tools. He saw the CAS-S as a representation on par with a table and graph and wanted students to reflect on the strengths and weaknesses of each of these tools when solving mathematical problems. In the last lesson, Chris was much more concerned that students practice mathematical procedures by hand as this helped them to better understand procedures. Nonetheless, Chris did allow students to use CAS-S when problems involved investigating mathematical ideas. Occasionally, he also asked students to reflect on the pros and cons associated with the use of CAS-S technology. The implications of these results for university level technology courses for prospective teachers will be discussed.

**Key Words:** prospective secondary mathematics teachers; computer algebra systems; symbolic manipulation, textbooks

Davis.pdf

**Norbert Domes, König-Heinrichs-Schule Fritzlar, Germany**  
*Teaching Mathematics with CAS SAGE*

1. Quick overview of Sage and the Sage notebook. Sage offers a wide range of mathematical topics including nearly 100 open-source packages, integrating their functionality. It provides an extended help, a lot of tutorials and examples. Using the programming language Python Sage becomes a flexible and extendible tool. Sage offers various ways of visualization, animated graphics, interactive controls. Therefore Sage is a powerful tool for teaching mathematics. The Sage notebook provides an user interface in a web browser, including user management as well as creating, sharing and publishing worksheets (the Sage notebook as collaboration framework). Beside the Sage notebook there are interesting related projects: Sage Cloud Server and Sage Cell Server.

2. Examples of teaching mathematics using Sage on (advanced) secondary school level. Presentation of worksheets: visualization, algorithmic problem solving, approximate solutions, challenging tasks.

3. Discussion: Issues of using Sage on school level. Sage is a huge and complex system, help and examples are mainly on university level, error messages seem to be confusing, there are some problems of unexpected behavior. Very likely beginners (students as well as teachers) may strand soon. Thus there is a need of specific examples and tutorials providing guidance for a successful use of Sage in school.

Web: [www.sagemath.org](http://www.sagemath.org); [nb.mathe-mit-sage.de](http://nb.mathe-mit-sage.de)

Keywords: CAS, Sage, Notebook

Domes.pdf

**Hamza Duhaini, Academic Bridge Program, Qatar**  
*Changing How Math is Taught and Learned Using*

High school students in Qatar are facing many challenges in learning Mathematics for many reasons some of which are:

1. Lack of motivation and interaction when working in Math whether at home or in class.
2. Memorizing instead of understanding mathematical principles
3. Difficulty with vocabulary and terminology of mathematics, understanding directions and explanations, or translating word problems - particularly when it involves irrelevant or out of sequence information;
4. Trouble learning or recalling concepts, definitions and meanings of abstract terms;
5. Difficulty reading texts to direct their own learning and communicating mathematics, including asking and answering questions;
6. Emotional block to mathematics originally due to Math anxiety.

In this session I will show a power point presentation about the features of MyMathlab and what it offers students and instructors to address some of these challenges.

With MyMathLab students can:

- » Practice and learn mathematics when and where they want and at their own pace.
- » Work through unlimited tutorial exercises correlated to the exercises in their textbook and get immediate feedback on their answers.
- » Receive a personalized study plan to diagnose areas where they need to practice.
- » Access a multimedia textbook with links to learning aids, such as animations and videos.
- » Use online tools, such as a discussion board or virtual classroom, to communicate with the teachers and other students.

- » Feel as if they have their own personal tutor, always available, and dedicated to their success.

With MyMathLab instructors can:

- »» Deliver all or a portion of their course online, whether the course is based in a lab, in a traditional classroom, or taught in a hybrid format.
- » Create and assign online homework, quizzes and tests that are automatically graded and tightly correlated to their textbook.
- » Manage and track students' results in a powerful online gradebook designed specifically for Mathematics and Statistics.
- » Customize their courses depending on their syllabus and their students' needs.
- » Use a variety of communication tools to create a supportive online community for their students.

Observations

- » The percentage of students earning As and Bs in their Math courses have increased since we introduced MyMathLab into our curriculum.
- » The success rate of those students in their subsequent course at their college also increased.
- » Students are spending more time practicing mathematics this year than any previous years.
- » Teachers are tracking students' performance much better and providing extra help for struggling students.
- » Teachers are interacting more with their students and are well aware of their strengths and weaknesses.

Keywords: MyMathLab, Homework, Quizzes and Tests, Study Plan, Grade Book.

Duhaini.pdf

## **Gerhard Egger, PH-NOE, BG/BRG Stockerau, Austria**

### *Gleichförmige Bewegungen – ein Unterrichtskonzept für die 9. Schulstufe*

Beim Thema „gleichförmige Bewegungen“ sollen gezielt Schüler/innen-Kompetenzen in verschiedenen Handlungsbereichen aufgebaut werden. Schwerpunkte werden in den Bereichen Darstellen/Modellbilden und Interpretieren liegen.

Vor allem soll an Alltagserfahrungen der Schüler/innen angeknüpft werden, und es sollen Eigentätigkeit und kooperatives Arbeiten gefördert werden. Von Schüler/innen-Gruppen erstellte Beispiele sollen in der eigenen Klasse präsentiert werden und als Übungsmaterial zur Verfügung stehen. Besonderes Augenmerk soll dabei auf Präsentationskompetenz und Kommunikation über Mathematik gelegt werden. Die Schüler/innen sollen auch zu konstruktivem fachlichen Feedback befähigt werden.

Die Beispiele werden in entsprechend gestalteten Textfiles zur Verfügung gestellt, die Lösungen in ppt-Präsentationen unter Verwendung von GeoGebra. Zum Abschluss werden die Gruppenergebnisse der Klasse vorgestellt.

Der Vortrag präsentiert das Unterrichtskonzept und integriert die Schüler/innen: Sie präsentieren selbst ihre Arbeitsergebnisse und stehen anschließend für Fragen zur Verfügung.

Variante: Sollte aus Zeitgründen die Teilnahme von Schüler/innen nicht möglich sein, werden im Unterricht erstellte Videos präsentiert.

Egger.pdf

**Wade Ellis, West Valley College,  
William C. Bauldry, Appalachian State University, USA**  
*Assessment with Access to a Computer Algebra System*

Assessment of student written responses to mathematical questions in the form of expressions, equations, or graphs should offer quality feedback. This presentation will describe and demonstrate two packages that allow instructors and authors to create problems requiring student written responses and that can provide quality feedback for these responses. These early-times packages will illustrate the issues and possibilities of such packages. The underlying CASs are TI-Nspire and Maple.

**Rainer Heinrich, Germany**  
*Zeitgemäßer Mathematikunterricht und zentrale Matura?  
Passt das zusammen?*

In Deutschland gibt es inzwischen in 15 von 16 Bundesländern zentrale schriftliche Abiturprüfungen. Der Autor selbst war seit 1992 bis 2002 Leiter der Autorengruppe für die schriftliche Abiturprüfung im Fach Mathematik. Durch einen grundsätzlichen Wandel der Vorstellungen von Mathematikunterricht hin zu einer stärkeren Orientierung auf den Erwerb von Kompetenzen, aber auch durch die Verfügbarkeit neuer Medien haben sich Unterricht und zentrale Prüfungen inzwischen verändert.

In Deutschland haben in den letzten 10 Jahren viele Länder das Zentralabitur eingeführt. Darüber hinaus haben die Länder Bayern, Hamburg, Niedersachsen, Mecklenburg-Vorpommern, Sachsen und Schleswig-Holstein im Jahre 2014 sogar erstmalig ein länderübergreifendes Zentralabitur eingeführt.

In diesem Zusammenhang waren viele Fragen zu beantworten, war nach Lösungen zu suchen:

- \* Welche Rolle spielt Technologie im Unterricht und in Prüfungen?
- \* Wie müssen die Schüler Lösungswege dokumentieren?
- \* Wie sollte die Struktur einer Prüfung gewählt werden.
- \* Welche Rolle spielen hilfsmittelfreie Prüfungsteile?
- \* Was ist künftig eine „leichte“ Aufgabe?
- \* Welche Inhalte sollten in einem Mathematikunterricht mit Technologie berücksichtigt werden?

Im Vortrag sollen einige dieser Fragen anhand verschiedener Beispiele von Abituraufgaben untersucht werden.

Heinrich Deutsch.pdf

**Wilfried Herget, Universität Halle-Wittenberg, Germany**  
*Funktionen – immer gut für eine Überraschung*

Ziel des Vortrags ist es, (wieder) bewusst zu machen, welch weites Spektrum an Überraschendem das Thema Funktionen bietet, und zu ermutigen, solche Gelegenheiten im Unterricht bewusst zu pflegen, und an den konkreten Beispielen aufzuzeigen, dass und wie Überraschendes im Mathematikunterricht wirkungsvoll zu nutzen ist – mit und ohne Rechner.

Herget.pdf

**Hans-Dieter Janetzko, HTWG Konstanz, Germany**  
*CATO – a general user interface for CAS*

Oftentimes, scientists, engineers, and students apply computer algebra systems only occasionally. Rare usage of grammar and syntax is a big obstacle to them. Therefore, the author proposes a general user interface for many computer algebra systems (currently Maple, Mathematica, Maxima, MuPAD, MuPAD inside MATLAB, and Yacas) named CATO.

The interaction is independent of the used computer algebra system, and CATO implements various concepts for a guided use of computer algebra. Commands with more than one parameter have a two dimensional input structure, parameters being explained and the right kind of separators and brackets are set by CATO. Selecting and adjusting options for commands is always similar. Additionally, the help contains for each command often used phrases. Currently, CATO is based on German, but the proposed concept could be easily extended to other languages.

In CATO, commands are combined to packages simplifying the access and selection. Commands may be contained in more than one package, as they might belong to more than one concept. The „definition of a vector“, for instance, is contained in both the „definition“ and the „linear algebra“ package. Accessing the most recently used commands is possible by lists storing the history of command invocations. Users are also able to define such a list as a new package and can export and import these as their own packages.

[Janetzko.pdf](#)

**Josef Lechner, Austria**  
*Verschiedene Zugänge zur Zahl e*

Die Euler'sche Zahl  $e$  ist eine der merkwürdigsten Zahlen, die in der Mathematik auftreten. So gibt es auch kaum ein wichtiges Gebiet der Schulmathematik (der Oberstufe) in dem sie nicht direkt oder indirekt in Erscheinung tritt.

Während  $e$  im Unterricht meist über den Grenzwert  $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n$  eingeführt wird, ermöglicht

der Einsatz der Technologie bereits vor der Differentialrechnung eine Einführung, bei der sich die Bedeutung der Zahl  $e$  für Schülerinnen und Schüler besser erschließt.

Darauf aufbauend soll auf verschiedene Problemkreise eingegangen werden, die mit der Zahl  $e$  in Verbindung stehen: Die Differenzen- und die Differentialgleichung des exponentiellen Wachstums und ihre Lösungen in Form der geometrischen Folge und der Exponentialfunktion. Damit in Zusammenhang steht wieder das Problem der Gemeinsamkeiten bzw. Unterschiede zwischen diskreten und kontinuierlichen Wachstumsformen.

Auch in der Differentialrechnung dreht sich sehr vieles um  $e$ , etwa: Welche Funktion ist gleich ihrer Ableitung? Wie sieht das Polynom aus, das  $e^x$  bestmöglich annähert?

Ein besonders reichhaltiges Leben entfaltet  $e$  bzw.  $e^x$  im Komplexen. Dort ist  $e$  nicht nur aufs Engste mit dem Kreis verbunden (wie sich in einer der schönsten Formeln der Mathematik zeigt), sondern auch periodisch.

Gerade der Einsatz der Technologie ermöglicht es, viele Aspekte der Zahl  $e$  bzw. der Exponentialfunktion  $e^x$  besser einsichtig zu machen und so Zusammenhänge aufzuzeigen, die auf Schulniveau sonst schwer zu vermitteln sind.

[Lechner-Zahl-e.pdf](#)

## **Carl Leinbach, USA**

### *Representing Numbers as Continued Fractions and a TI-Nspire Package to do some Basic Continued Fraction Arithmetic*

From birth most human beings are confronted by the number 10. In general, we have ten fingers and ten toes. It was only natural that we developed decimal numbers for doing our basic calculations. When computers came along with their binary states (over simplified as a switch being on and off) binary arithmetic came into its own. (One wonders, do cats use a base twelve arithmetic when catching mice?) However, decimal and binary arithmetic have their limits when it comes to doing calculations that require very accurate approximations for certain results. Accurate decimal approximations may require the use of numerous digits and computing systems contain only a finite model of the real numbers. Continued Fractions are an alternative representation for numbers that are generally able to compactly represent approximations to real numbers for a desired accuracy. Computer Algebra Systems have long used continued fractions while implementing the approximation operation when requested by a user.

Continued fractions have their roots in the so-called Euclidean Algorithm for finding the greatest common divisor of two integers. In fact, finding the continued fraction representation of a rational number follows the steps of this algorithm exactly. However, they did not become an object of mathematical study until the work of Brouncker and Wallis in the late 17<sup>th</sup> century. An impediment to their growth in popularity was the lack of an efficient system of doing arithmetic with this representation of the real numbers. In 1972 William Gosper developed an efficient system for doing arithmetic with continued fractions and show them that numbers can be represented more efficiently as continued fractions. For example,  $\sqrt{17} = 4.12310562561\dots$ , can be approximated as  $[4; 8, 8, 8, 8] = 17684/4289 = \mathbf{4.123105619\dots}$  that has a much smaller denominator than the decimal representation having the same accuracy.

In this presentation, a small package for use in classroom exercises to introduce students to this different way of viewing real numbers and using continued fractions, thus, expanding their view of number representations and their uses, will be discussed. It will deal with converting rational numbers to continued fractions as well as the reverse process. It will also include an algorithm for expressing square roots of integers as, sometimes infinite, periodic continued fractions. Also included will be Gosper's algorithms for doing arithmetic with continued fractions. Time permitting, other topics such as the expansion of  $\pi$  and  $e$  as infinite continued fractions will be presented. A TI- Nspire package containing these results developed by the author will be previewed. It will also be made available for sharing with participants.

[Leinbach.pdf](#)

## **Andreas Lindner, PH Upper Austria, Austria**

### *GeoGebra 3D*

The new version of GeoGebra 5.0 provides a module for dynamic geometry in 3D. With this new module it is possible to create constructions of spatial geometry in connection with planar geometry. The procedure is analogous to the usual work in the graphics window used previously and requires no change in the operation.

The created geometric objects are displayed as usual in algebraic form. In combination with the computer algebra system GeoGebraCAS thus a geometrical representation and a mathematical treatment of the problem are possible.

In the lecture examples from analytical geometry, calculus and some physical applications are presented.

[Lindner.pdf](#)

**Robert Märki, T3-Switzerland**

*Differential Equations and Dynamical Systems, a dynamic approach  
with TI-NspireCAS*

A vast majority of applications of calculus in different sciences consists of models with systems of differential equations e.g. dynamical systems, as any perusal of the current literature will indicate. In addition, as Henri Poincaré showed, the fundamental laws of nature are, from a mathematical point of view, differential equations. Therefore, in our opinion, differential equations and dynamical systems should be at the center of every introductory calculus course and this can be accomplished through the application of technology. It is now possible to solve differential equations numerically and graphically with little manual effort, allowing them to take their rightful place in an introductory course. The development of conceptual understanding rather than algebraic technique is important. We designed an introductory calculus course based on these principles. We will show, how this course is structured. It begins with a simple model (Newton's law of cooling) which leads -at first more intuitively- to the fundamental concepts of derivative (rate of change), differential equation, Euler's Method, model and dynamical system. We introduce the derivative in the original and general sense as flux and not through the traditional slope of a tangent line. The central chapters are dedicated to differential equations and dynamical systems, introducing the graphical method of state plane analysis. With the TI-Nspire technology direction fields and trajectories can be represented graphically and thanks to the sliders it is even possible to experimentally investigate the impact of the variation of the parameters. This can enhance a deep understanding of the systems under consideration. We will demonstrate the power of these methods with some impressive examples. With the focus on dynamical systems the great significance of calculus can be far better demonstrated than with the traditional approach to introducing calculus.

[MaerkiDiffEqu.pdf](#)

**Douglas B. Meade, University of South Carolina, USA**

**Philip B. Yasskin, Texas A&M University, USA**

*Maplets for Calculus: A Model for Multi-Use Mathematical Software*

Maplets for Calculus (M4C: <http://m4c.math.sc.edu/>) is a collection of more than 200 customized user interfaces for precalculus, calculus and differential equations. Unlike most online homework systems that look only for final answers, each Maplet checks each step in the solution process and gives help as needed. A unique feature of the M4C is that the same resources are equally useful for instructors and students. They can be used as demonstrations in lecture to demonstrate a new concept, as a group activity in a lab setting, as a drill-and-practice tutor by individual students, and as a graded activity when assessing student mastery of the concept. For example, when introducing the concept of volume as a definite integral of cross-sectional area, the same 3D animations that an instructor would use in lecture are available to students when they are working to complete their homework. In addition, the separate Maplet for the methods of disks, washers, and shells, uses a parallel approach building upon what they have just learned. The interface includes pedagogical features designed to help students develop mastery; these features are disabled when used in a graded setting. Demonstrations of these two Maplets will be included in this presentation. The use of a CAS is essential. It is used to algorithmically generate an almost endless collection of problems, to determine if responses are mathematically consistent even if they might be entered in a somewhat different form, to produce feedback based on the specific response entered, and to produce appropriate graphics (2D, 3D, animation, and stereo). In spite of all of the benefits of the M4C, a number of technical obstacles have arisen. For example, Maplets are rendered using Java which limits their accessibility. Some users have difficulties with the syntax for entering mathematical expressions. To address these concerns the authors have worked to re-create the M4C using JavaScript and HTML5; the new resources are called MYMA (Meade- Yasskin Math Apps) Lessons. An early result of these efforts is MathLex, a palette-based mathematical entry system written in JavaScript. The presentation will include demonstrations of MathLex, and prototypical Lessons.

Meade.pdf

**Dennis Pence, Western Michigan University, USA**

*Explorations with the Barycentric Formula for Polynomial Interpolation*

We will explore graphing calculator and other implementations of the barycentric formula for polynomial interpolations. These are related to some of the chebfun package implementations of these ideas for Matlab provided by L. N. Trefethen. Interestingly, Peter Henrici provides some of this in his 1982 book, *Essentials of Numerical Analysis with Pocket Calculator Demonstrations*, but it seems to have been little noticed.

There are many ways to express polynomials. Some of the ways have advantages for theoretical work (say for interpolation), and some of the ways have advantages for computation, particularly on computers and calculators. I have been teaching for over 30 years that the *Lagrange form*

$$p(x) = \frac{y_0(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} + \frac{y_1(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} + \cdots + \frac{y_n(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}$$

is very nice for theoretical work with interpolation, but it does not seem all that practical for computer implementation.

Further, following the example of Runge, I have also warned my student against the practice of trying polynomial interpolation with high degree polynomials. Runge's example using equally-spaced nodes is still standard in almost all numerical analysis textbooks. But several

recent papers show the way, both to use the Lagrange form and to use high-degree polynomial interpolation for practical computation.

The first step is to consider the barycentric formula for this Lagrange form. Obviously we can precompute parts of the Lagrange form. Let the weights be

$$w_i = \frac{1}{(x_i - x_0)(x_i - x_1) \cdots (x_i - x_{i-1})(x_i - x_{i+1}) \cdots (x_i - x_n)}, \quad i = 0, 1, \dots, n$$

and

$$l(x) = (x - x_0)(x - x_1) \cdots (x - x_n)$$

which is usually called the *nodal polynomial*. The first *barycentric* formula is then

$$p(x) = l(x) \sum_{i=0}^n \frac{w_i y_i}{(x - x_i)}.$$

Trefethen claims that this may go back to Jacobi's thesis. Notice that this (now rational expression) is technically not defined at the exact nodes  $x_i$ , but we already know the value of the polynomial here, namely the interpolation values  $y_i$ . Next we know that polynomial interpolants are unique, so the Lagrange form that interpolates the constant function 1 will give

$$1 = l(x) \sum_{i=0}^n \frac{w_i}{(x - x_i)}.$$

Dividing the last two formulas, we get the second barycentric formula

$$p(x) = \frac{\sum_{i=0}^n \frac{w_i y_i}{(x - x_i)}}{\sum_{i=0}^n \frac{w_i}{(x - x_i)}}.$$

This is the formula that we can nicely compute in a simple loop for any  $x$ , where we accumulate the sums in the numerator and denominator separately, and then divide after the loop is completed.

We will briefly review what is known about the weights for certain choices of nodes. Then we will consider implementations of these ideas in certain environments, including graphing calculators. This will be based on some of the ideas in the *chebfun* package for Matlab, given to us by L. N. Trefethen and his colleagues at Oxford University. Interestingly, these barycentric formulas can be found in the 1982 book, *Essentials of Numerical Analysis with Pocket Calculator Demonstrations*, by the Swiss mathematician Peter Henrici. Unfortunately this later book did not receive as much attention as his earlier *Elements of Numerical Analysis* where the barycentric formula does not appear. If you check on the *Mathematical Genealogy Project*, you will find that I am one of the 270 descendants of Peter Henrici.

## **Roland Pichler, Austria**

### *Mathcad im Unterricht und bei der neuen Reife- und Diplomprüfung*

Anhand von ausgewählten Unterrichtsaufgaben werden unterschiedliche Einsatzmöglichkeiten dieser Mathematiksoftware gezeigt. Dabei werden sowohl die numerischen und graphischen Möglichkeiten als auch die Möglichkeiten des Einsatzes als CAS aufgezeigt.

Es wird das Augenmerk in erster Linie auf den Unterrichtseinsatz gelegt, ohne dabei auf spezielle technische Feinheiten von Mathcad einzugehen. Durch den regelmäßigen Einsatz im Unterricht ergibt sich daraus automatisch die Verwendung bei der schriftlichen Reife- und Diplomprüfung, wobei sich vor allem die Möglichkeit der hervorragenden Dokumentation von Mathematikarbeiten als sehr vorteilhaft erweist.

Ein kurzer Ausblick auf einige spezielle Anwendungen im Mathematikunterricht runden den Vortrag ab.

[Pichler\\_pdfs.zip](#)

## **Rein Prank, Marina Lepp, University of Tartu, Estonia**

### *Conceptualizing a Pedagogical CAS for Algebraic Manipulation of Expression*

In this paper we define Pedagogical CAS as an exercise and assessment environment for exercises on numeric, fractional and algebraic expressions, equations, inequalities and equation systems. We assume that the student solves tasks step by step, performing arithmetic and algebraic operations. The system checks correctness of the student's steps, provides feedback about errors and requires correction, records the student's solutions and information about errors, is able to provide hints, has tools for visualizing recorded information and for automated assessment.

The paper describes our ideas on the following issues:

- Solution interface and working modes for students,
- Error diagnostics,
- Features and role of an automated Solver,
- Using random generation of expressions,
- Features and user interface of a Teacher Program for preparing tasks, analyzing the work of one student and groups of students, preparing tests with automated grading,
- Database and server architecture of the system, integration with course administration systems (like Moodle).

The paper is based on our experience with the programming and use of the interactive exercise environment T-algebra together with some supplementary programs but also on experience with a package for computerized exercises for an introductory course in Mathematical Logic.

[Prank.pdf](#)

## **Tom Reardon, USA**

### *Integrate iPad Math & Presentation into your 6-12 Math Classes*

Take photos with your iPad and immediately model the curves with an equation or a geometric figure – on your iPad! Obtain and “play” over 1000 FREE interactive classroom ready activities – including teacher notes and solutions for grades 6 – 12. Participants will learn how to cleverly utilize these apps: Reflector, Splashtop, Dropbox, and TI-Nspire. All documents referenced will be made available. If you have one iPad or a class set, see how to use this power.

Learn the power of the different apps. See how to use Reflector software on your computer in conjunction with the Airplay app on your iPad so that you can wirelessly project what is on your iPad through your projector onto the screen or interactive whiteboard. Experience how to use the Splashtop Whiteboard app that will allow you to control your computer desktop wirelessly. Also be able to make annotations on your iPad screen that will appear on your desktop software apps. See how to utilize the Dropbox app and share files quickly and easily with colleagues and students. And use all the powers of TI-Nspire CX CAS in a touch environment with the TI-Nspire app for iPads. Dynamic Geometry, CAS, graphing, spreadsheets, data and statistics – integrate all these apps into one on your iPad. An excellent teaching and learning tool.

[Reardon\\_Integrate.pdf](#)

## Tom Reardon, USA

### *CAS and Dynamic Geometry Activities that Integrate Algebra & Geometry: Investigate, Discover, Prove*

This is a cool way to learn ... and teach: have your students investigate a geometry activity – hands-on, interactively, self-paced; then have them test and generate their hypotheses; and then have them prove their results using CAS. They are still doing the thinking, the logic, the investigating, the problem solving, but let CAS assist them to do the difficult mathematics. Lots of great mathematics in these activities. And some surprising, but fascinating results.

We will do as many investigations as time permits using the TI-Nspire CX CAS handheld. Here are the activities that I will choose from:

1. The Square Problem. Given a square with both diagonals drawn.  
Find the midpoints of the 4 "half-diagonals".  
Draw the 4 segments connecting each vertex with a different midpoint.
  - a) Of what type is the resulting figure?
  - b) What is the relationship between the area of the original square and the resulting figure?
2. Midpoint Polygons an Investigation. Given a regular polygon, calculate its area and perimeter. Then construct the polygon by connecting consecutive midpoints of the sides of the original polygon. Is there a relationship between the:
  - a. ratios of the perimeters of the two polygons?
  - b. ratios of the areas of the two polygons?Is so, state and prove those relationships.
3. Other Square Problem. Given a square. Construct the midpoints of each side. Draw segments connecting each vertex to the 2 midpoints not on the sides that contain the vertex.
  - a) What is the best descriptor of the type of polygon that is formed.
  - b) Is there a relationship between the area of the square and the area of the polygon?  
If so, what is it?

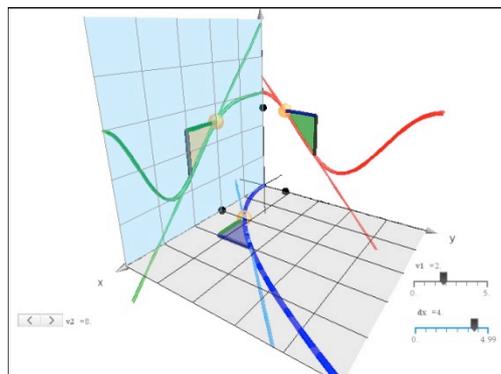
[Reardon\\_Dynamic\\_Geom\\_CAS.pdf](#)  
[Reardon\\_Dynamic\\_tns.zip](#)

## **Geneviève Savard, ETS Montréal, Canada**

### *3D & TI-NspireCAS: What is available, What is Missing and How to adapt*

In our multivariable calculus classes, 3D graphs and animations are used to present new concepts and to verify results. Last year's OS 3.2 marked a major improvement in the TI Nspire CAS graphing capabilities by introducing 3D parametric plotting. However, with its restricted functionalities, the 3D menu items are not yet equivalent to what is available in the 2D plot environment.

In this presentation, we will illustrate how we can work around the 3D plotting limitations to access important geometric objects that are available in the 2D environment e.g. points, vectors or curve with thickness. We will also present a library that was created by colleagues in order to facilitate 3D-object plotting. Examples used in our calculus classes will be provided: partial derivative, tangent plane, normal vector, and chain rule.



Keywords: 3D graphs, calculus, 3D library for TI Nspire, parametric plotting

[Savard.pdf](#)  
[Savard\\_geo3d.pdf](#)

## **Mazen Shahin, Delaware State University, USA**

### *Explorations of Mathematical Models in Life Sciences with Maple*

In this lecture we will share with the participants samples of the instructional materials for an interdisciplinary course on mathematical models of biology with the CAS, Maple. The main goal of the activities is to help students explore and discover mathematical concepts and use these concepts in building and analyzing mathematical models of life science disciplines such as biology, ecology, and environmental sciences.

In this lecture, we introduce the modeling of real life situations with difference equations and matrices using Maple. We will emphasize the use of graphical and numerical techniques, rather than theoretical techniques, to investigate and analyze the behavior of solutions of the mathematical models. We will investigate interesting linear and nonlinear models from diverse life science disciplines.

We utilize a discovery pedagogical approach. To introduce a concept, first we investigate a model numerically and/or graphically and recognize a pattern or certain properties that characterize that concept. Then we give a definition of the concept with examples and applications. For example, to introduce the eigenvalues and corresponding eigenvectors of a square matrix we investigate an age-structure population model with different initial population vectors that lead to a visualization of an eigenvalue and corresponding eigenvectors. Then the definition of the eigenvalues and eigenvectors are introduced and some properties are discussed.

Why modeling with difference equations and matrices? Difference equations represent a very sophisticated and powerful mathematical tool to model a wide range of real life discrete time situations in diverse areas, including the life sciences. And matrices provide an excellent tool in modeling linear problems. Moreover, these powerful tools do not require a sophisticated mathematics background, being accessible to anyone who has successfully completed high school algebra or college algebra.

Why do we use Maple?

All the models presented require the use of computers. For example, in order to investigate and analyze a model it is often required to iterate the difference equation(s) or the matrix difference equation(s) that represent the model and graph it. Sometimes it is required to find the eigenvalues and the corresponding eigenvectors in order to investigate the long-term behavior of a dynamical system. In other instances, in order to investigate the sensitivity of a dynamical system to certain parameters, it is required to change the parameters of the dynamical system and find the corresponding numerical solutions. All these computational activities require a software that is easy to learn and to use. Maple is a user friendly and powerful mathematics software with excellent graphing capabilities. The use of Maple frees students from tedious calculations. This allows them to focus on translating a problem into mathematical notation, finding a solution, interpreting the numerical and the graphical information provided, and then making conjectures and writing about their findings and observations. With the use of Maple the students focus on building and analyzing the models.

### **Karsten Schmidt, Schmalkalden University of Applied Sciences, Germany** *Lösung linearer Gleichungssysteme am PC – „klassisch“ vs. „modern“*

Wir beschäftigen uns mit der Lösung linearer Gleichungssysteme mit  $m$  Gleichungen in  $n$  Unbekannten unter Verwendung eines Computer-Algebra-Systems (*DERIVE*).

Zunächst veranschaulichen wir mit 3 Beispielen für den Spezialfall  $m=n=2$  die möglichen Situationen graphisch:

- 1) Das Gleichungssystem hat genau eine Lösung: zwei Geraden schneiden sich in einem Punkt.
- 2) Das Gleichungssystem hat unendlich viele Lösungen: zwei Geraden liegen übereinander.
- 3) Das Gleichungssystem ist nicht lösbar: zwei Geraden liegen parallel zueinander.

Als weiteren Spezialfall untersuchen wir lineare Gleichungssysteme mit  $m=n=3$  graphisch.

Außerdem werden die Beispiele algebraisch gelöst, was wegen der Nutzung der SOLVE-Funktion des Computer-Algebra-Systems besonders einfach ist.

Ein lineares Gleichungssystem mit  $m$  Gleichungen in  $n$  Unbekannten lässt sich in der Form  $Ax = b$  mit der (bekannten)  $m \times n$ -Matrix  $A$ , dem (bekannten)  $m \times 1$ -Vektor  $b$  und dem (unbekannten)  $n \times 1$ -Vektor  $x$  schreiben. Nun können wir die Beispiele matrixalgebraisch lösen, verfügen also über eine dritte Methode zur Lösung linearer Gleichungssysteme. Allerdings lässt sich nur die eindeutige Lösung (Fall 1) direkt mittels  $x=(A^{-1})b$  berechnen; in den Fällen 2 und 3 existiert die Inverse der Matrix  $A$  nicht. Daher wird eine „moderne“ vierte Lösungsmöglichkeit betrachtet. Zu diesem Zweck ist die Berechnung der so genannten Pseudoinversen der Matrix  $A$  erforderlich. Da wir *DERIVE* verwenden, das nicht über eine eingebaute Funktion zur Berechnung der Pseudoinversen verfügt, muss dafür eine Bibliothek hinzugeladen werden. Wenn (die  $n \times m$ -Matrix)  $P$  die Pseudoinverse von  $A$  bezeichnet, lässt sich jedes lineare Gleichungssystem in einem einfachen zweistufigen Verfahren lösen:

1. Es muss überprüft werden, ob  $APb=b$  ist; falls dies nicht der Fall ist, ist das Gleichungssystem nicht lösbar (Fall 3).
2. Sofern  $APb=b$  ist, lautet die allgemeine Lösung  $x=Pb+(I-PA)z$ , wobei  $z$  ein beliebiger  $n \times 1$ -Vektor ist (Fälle 1 und 2).

**Hildegard Urban-Woldron, University College for Teacher Education Lower Austria**

*Comprehension of calculus concepts based on motion sensor data*

Results from physics education research indicate that laboratory activities using a motion sensor can supply a powerful setting for addressing students' difficulties in interpreting graphs of kinematics variables. Furthermore, graphs are also fundamental mathematics tools, whose application requires various types of mathematical competencies. The focus of the paper is on activities to develop conceptual understanding of fundamental calculus concepts, specifically instantaneous rate of change and accumulation of area under a curve by introducing motion sensor devices to the mathematics classroom. Beyond enhancing the teaching of mathematics by making it more realistic, alive and accessible, the motion sensor device enables users to get real time  $x(t)$  and  $v(t)$  graphs of a moving object or person. Consecutively, these graphs can be productively used to introduce differentiation and integration on visual levels. In this context, the author presents examples based on real experiments where functions can be viewed from their characteristic behaviour-over-time/rate-of-change patterns, explaining and discussing the steps of such an approach in further detail. In conclusion, the technology use in the classroom challenges teachers to re-examine teaching approaches in order to fully exploit the potential of the resources. Calculus, as the mathematics of change and accumulation of quantity, is conceived as a context in which the use of data from real experiments can significantly contribute to conceptual understanding of mathematics concepts.

Urban-Woldron.pdf

# Short Lectures

**Gabriel Aguilera, Jose Luis Galan, Mara Angeles Galan, Yolanda Padilla, Pedro Rodriguez, University of Malaga, Spain, Ricardo Rodriguez, Technical University of Madrid, Spain**

*Advanced Techniques to compute improper integrals using a CAS*

Let us consider the following types of improper integrals:

$$\int_0^{\infty} f(t) dt \quad ; \quad \int_{-\infty}^0 f(t) dt \quad \text{and} \quad \int_{-\infty}^{\infty} f(t) dt$$

Let  $F$  be an antiderivative of  $f$ . The basic approach to compute such integrals involves the following computations:

$$\int_0^{\infty} f(t) dt = \lim_{m \rightarrow \infty} \int_0^m f(t) dt = \lim_{m \rightarrow \infty} (F(m) - F(0))$$

$$\int_{-\infty}^0 f(t) dt = \lim_{m \rightarrow -\infty} \int_m^0 f(t) dt = \lim_{m \rightarrow -\infty} (F(0) - F(m))$$

$$\int_{-\infty}^{\infty} f(t) dt = \int_{-\infty}^0 f(t) dt + \int_0^{\infty} f(t) dt \quad \text{or, in case of convergence}$$

$$\int_{-\infty}^{\infty} f(t) dt = \lim_{m \rightarrow -\infty} \int_{-m}^m f(t) dt = \lim_{m \rightarrow \infty} (F(m) - F(-m)) \quad (\text{Cauchy principal value})$$

But, what happens if an antiderivative  $F$  for  $f$  or the above limits do not exist?

For example, for

$$\int_0^{\infty} \frac{\sin(at)}{t} dt; \quad \int_0^{\infty} \frac{\cos(at) - \cos(bt)}{t} dt \quad \text{or} \quad \int_{-\infty}^{\infty} \frac{\cos(at)}{t^2 + 1} dt$$

the antiderivatives can not be computed. Hence, the above procedures cannot be used for these examples.

In this work we will deal with advance techniques to compute this kind of improper integrals using a CAS. Laplace and Fourier transforms or Residue Theorem in Complex Analysis are some advance techniques which can be used for this matter.

We will introduce the file `ImproperIntegrals.mth`, developed in DERIVE 6, which deals with such computations. Some CAS use different rules for computing integrations. For example Rubi system, a rule-based integrator developed by Albert Rich (see <http://www.apmaths.uwo.ca/~arich/>), is a very powerful system for computing integrals using rules. We will be able to develop new rules schemes for some improper integrals using `ImproperIntegrals.mth`. These new rules can extend the types of improper integrals that a CAS can compute.

Keywords: Improper integrals, DERIVE, CAS, Laplace Transform, Fourier Transform, Residue Theorem

[Galan\\_Advanced.pdf](#)  
[Galan\\_Advanced.dfw.pdf](#)

**Gabriel Aguilera, Jose Luis Galan, Maria Angeles Galan, Yolanda Padilla, Pedro Rodriguez, University of Malaga, Spain, Ricardo Rodriguez, Technical University of Madrid, Spain**

*A stepwise CAS Course for solving First-Order Partial Differential Equations*

Partial Differential Equations (PDE) is a very important topic in advance Mathematics for Engineering. The three main first-order PDE problems that a basic course must deal with are:

1. **Pfaff Differential Equations**, which consists on finding the general solution for:

$$P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$$

2. **Quasi-linear Partial Differential Equations**, which consists on finding the general solution for:

$$P(x, y, z)p + Q(x, y, z)q = R(x, y, z)dz \quad \text{where } p = \frac{\partial z}{\partial x} \text{ and } q = \frac{\partial z}{\partial y}.$$

3. **Lagrange-Charpit Method** for finding a complete integral for a given general first order partial differential equation:  $F(x; y; z; p; q) = 0$ .

In order to help the teaching and learning process of this topic, we have developed, using Derive 6 Cas, the file FOPDE.mth. The use of this file allows the user to solve these three problems (their general cases and the particular cases) stepwise. Since these three problems requires several steps for their resolution, the programs developed in FOPDE.mth show step by step all the resolution procedure providing in this way a powerful tool as a tutorial for teaching how to solve these types of equations. This way the use of Derive 6 is done as a PeCas (Pedagogical CAS) providing not only the final result but also all partial results.

On the other hand, in the resolution process of such equations, first-order Ordinary Differential Equations (ODEs) are needed. Therefore, FOPDE.mth loads the package FOODE.mth, which is part of the Derive package that was introduced in TIME 2010.

The programs contained in the file FOPDE.mth can be grouped within the following blocks:

- » **First-order ODEs:** separable equations and equations reducible to them, homogeneous equations and equations reducible to them, exact differential equations and equations reducible to them (integrating factor technique), linear equations, the Bernoulli equation, the Riccati equation.
- » **First-order differential equations and nth degree in y'.**
- » **Generic programs to solve first order differential equations.**
- » **Pfaff Differential Equations.**
- » **Quasi-linear PDE.**
- » **Lagrange-Charpit Method for First-Order PDE.**

Finally, we will state the conclusions obtained after using this file with our students and also some future work on this and other related subjects.

Keywords: ODEs, PDEs, Derive, Cas, Pedagogical Computer Algebra System (PeCas), Engineering

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Galan\_PDE2.pdf  
Galan\_PDE2.dfw

**Morenikeji A. Akanmu, Michael O. Fajemidagba, University of Ilorin, Nigeria**  
*The Use of Graphical Materials Teaching of Mathematics: Effects of Students' Understanding and Performance*

Mathematics teaching and learning process expectedly should portray an interactive and dynamic classroom experience capable of promoting mathematical understanding and sustaining learners' interest in anticipation of applying what they have learnt. This study, being quasi-experimental, therefore expands the empirical base for understanding learners' cognitive load memory capacity in relation to use of graphical materials on students' mastery of mathematics concept of Set Theory. The sample consisted of seventy-five (75) students from the selected schools in Egbeda, Oyo State. Three research hypotheses were tested using t-Test statistics. Evidence from field survey indicates that students were more enthusiastic in classroom because of the interactive nature portrayed by the graphical materials. Results also indicated that students taught with the aid of graphical materials, that is, the experimental group, had significantly higher mean gain score and it was recommended that, teachers should begin the use of graphical materials as opportunity to make the learning of mathematics easier and that, if properly harnessed, this aspect of instruction may serve as simulator helping learners construct meanings and develop their own understanding, ready to take on more complicated structures as they moves towards complex concepts in mathematics.

Keywords: Graphical Materials, Cognitive Load Memory, Performance, Students, Mathematics.

[Akanmu.pdf](#)

**Michel Beaudin, ETS Montréal, Canada**  
*NspireCAS and Laplace Transforms*

There is no built-in Laplace transform function in Nspire CAS but a library can be used to deal with this stuff - essentially the one written originally for TI-89. For more details about this library, our colleague Chantal Trottier wrote an interesting document with examples:

<http://seg-apps.etsmtl.ca/nspire/documents/transf%20Laplace%20prog.pdf>.

The library can also be downloaded there.

In this talk, we use this Laplace transforms package to automate some engineering applications, such as mass-spring problem and RLC circuit. Using animations of response to indicator function inputs, we will try to get a better understanding of what is a Dirac delta function. Also, the concept of convolution will be revisited.

Keywords: Laplace transforms, engineering applications, impulse functions.

[Beaudin\\_Laplace.pdf](#)  
[Beaudin\\_Laplace.tns](#)  
[Beaudin\\_ETS-Kit.zip](#)

**Wen-Haw Chen, Tunghai University, Taiwan**

*CAS in teaching Linear Algebra: From Diagnosis, Connection, Deepening to Application*

Linear algebra is a fundamental and important subject for students in science, engineering and management schools etc. Hence the study on the improvement of teaching and learning of linear algebra has been a crucial topic in undergraduate mathematics education. In this paper we present an experiment about integrate computer algebra system (CAS) in a Linear Algebra course carried out in the academic year 2013-14 at Tunghai University in Taiwan. The software is chosen to be MALAB and we divided the course into four stages: diagnosis, connection, deepening and application together with the use of CAS.

In the first stage “diagnosis”, we assess students’ prior knowledge (for examples, the notions about vector, inner product, matrix and determinant etc.) before processing the course. Secondly, we develop in the stage of “connection” the material for the bridging course according to the result of the diagnosis stage. Thirdly, we concern how to “deepen” the teaching of abstract theories. The final stage is to review whether students can “apply” learned knowledge to solve the practical problems designed by the characteristics of different fields. During the period of carrying out the four stages of the course, CAS will play the role to help instantaneous numerical and symbolical calculations: data collecting, analysis, exploration, and visualization; modeling, simulation, and prototyping; presentation graphics and animation in 2D and 3D; and application development.

Keywords: Linear Algebra, Computer Algebra Systems, Teaching methods, Undergraduate mathematics education, Curriculum

[Chen.pdf](#)

**Bernhard Eberl, PTC Academic Support, Austria**

*Mathcad in der Industrie*

Wir werden den 30 min Vortrag in Kooperation mit unserem lokalen Ansprechpartner Techsoft durchführen:

Der Vortrag wird die folgenden Themen umfassen:

1. Mathcad Allgemein – Overview
2. Mathcad Beispiele aus der Praxis:
  - a) Maschinenbau
  - b) Elektrotechnik
  - c) Mathematik
3. Wissensknowhow sichern und verwalten
- 4 Symbiose – Mathematik und Konstruktion

[Eberl.pdf](#)

**Ángel Homero Flores, Adriana Gómez, Xochitl Chávez, Colegio de Ciencias y Humanidades-UNAM, México**

*Using TI-NspireCAS in a Modelling Teacher's Training Course*

Using Mathematical Modelling has become a useful tool in teaching-learning mathematics at all levels. This is so because mathematical objects are seen from their very applications, giving them meaning from the beginning.

In this lecture we present some details on the development of a teacher's training course called *Modelling in the Teaching of Mathematics* for high school teachers in Mexico City metropolitan area. In that course we used TI-Nspire CX CAS calculators as a tool to pose and solve modelling problems in the context of the teaching model Learning Mathematics, Doing Mathematics. We are interested in highlight four aspects of the sessions:

- a) The kind of problems we can have in modelling as teaching-learning methodology.
- b) The use of calculators as facilitator tools in solving the problems.
- c) Teachers' attitudes and beliefs towards the use of this kind of technology in teaching high school students.
- d) Teachers' perceptions of the course and its usefulness in teaching mathematics.

We will illustrate our points looking at some examples of the teachers work during the course. In particular, it is interesting to see some solving strategies of the following problem:

*A goat is tied up at the corner of a rectangular cottage (3 x 5 meters) in the middle of a huge grass field. What's the area the goat can cover if the length of the rope is 4 meters? What's the length of the rope if the goat can cover a 50 squared meters area?*

*Find a mathematical model that describes the area covered in terms of the rope's length. Explain in full detail your model.*

Keywords: Mathematical Modelling, Teachers Training, CAS Calculators, Teachers Attitudes and Beliefs.

Flores\_using.pdf

## **Àngel Homero Flores, Jeanett López, México**

### *The Task we Do, the Software we Choose*

Some years ago in teaching of mathematics we had a certain amount of Dynamic Geometry software applications that offered some options in the design of teaching-learning activities. And we could choose the software according to the task and its objectives. This is so because the construction philosophy was different for each application, and thus the knowledge they foster.

Today, the trend is that only one of such applications will be at hand, and the only reason is that it is for free.

We stand that there is not a dynamic application better than the other, their use should depend on the task at hand and the learning objectives we want to achieve.

We will illustrate this with two teaching-learning activities developed with *GeoGebra* and *The Geometer's Sketchpad* and highlight the main features of each and the knowledge students learn and put into action using the software.

The two tasks are:

- A problem in which the solver has the need of adjusting a curve to a set of points without using a statistical tool.
- An activity aimed to foster the concept of iteration in fractals.

Keywords: Dynamic software; problem solving.

[Flores\\_The Task.pdf](#)

## **Martin Günzel, Czech Republic**

### *Working with nonverbal elements using DGS in the mathematics*

Non-verbal element as an independent means of communication plays an important role in the mathematics. We use it both ways - either to mediate curriculum content, or for expressing content using non-verbal elements. Visualization using non-verbal elements helps to better understanding the mathematical problems, especially in geometry. Geometry practically cannot do without visualization. The research on nonverbal elements and their interaction with human perception aims extensive research area called research on pictures. In this article we research on non-verbal elements in terms of their function as a visual representation and the aim of research is determine how DGS can improve pupil's work with nonverbal elements. Research is aimed at students of primary school in teaching geometry. In this article we try to answer the question of whether visual representations help in the transfer of learned using GeoGebra program.

[Guenzel.pdf](#)

**Roman Hašek, University of South Bohemia, Czech Republic**  
*Study of Historical Geometric Problems by Means of CAS and DGS*

Common use of the computer algebra system wxMaxima and the dynamic mathematics software GeoGebra to solve several geometric problems on conics and loci from an 18<sup>th</sup> century textbook will be presented. In particular examples will be shown of how the use of these programs helped the authors to understand the method that our predecessors used to deal with the conic sections together with the solving the problems.

The combination of DGS and CAS has proved its worth to the authors in solving of such problems. DGS provides us with the dynamic construction and gives us an idea of the solution, e.g. a locus shape, whereas CAS helps us to find an equation and to prove or disprove the generality of a quality that is captured by the dynamic figure. The use of the computer changed the old problems, which to our students were originally solved in a rather strange way, into attractive modern problems. The authors will present the possible utilization of these problems in the teaching of mathematics and the benefits of doing so.

Hasek.pdf

**Tim Horvat, Jasna Kos, Gimnazija Bežigrad, Slovenia**  
*Saving Private Goldbach*

A group of six students from Gimnazija Bežigrad, Ljubljana, Slovenia will present the performance “Saving Private Goldbach”. With this drama they will participate in the MATHeatre competition. This is an activity by the European Commission funded project “Learning Mathematics through new Communication Factors”. Tim Horvat, the 16-year-old author of the performance, presents the ethical dilemma about a computer being able to prove mathematical theorems and thus being able to replace the humans.

The performance is a result of extended work on mathematical proofs. Students learn about the sense of the proof in the first year of the school, when studying about the number theory. The first step is to understand that it is not enough to change variables with numbers. To prove something means to use axioms and deductive reasoning to explain why a statement is true without having to check every number. “Mathematicians aren’t satisfied because they know there are no solutions up to four million or four billion, they really want to know that there are no solutions up to infinity,” said Andrew Wiles. In the performance the famous Fermat’s last theorem and Goldbach’s conjecture are exposed. The performance was written by using information from the novel *Uncle Petros and Goldbach’s Conjecture* by Apostolos Doxiadis and the movie *The Fermat’s Last Theorem* by Simon Singh .

The performance goes as follows. In Act 1 Robert Stark informs Fred Euler about his achievement. He successfully launched “the most advanced computer in the human history which will push borders of maths to the unbelievable dimensions”. Two hours after setting up five Peano axioms the computer named Hal already proved the Fermat’s last theorem. There is a possibility that the constructor’s secret to wish to prove the Goldbach’s conjecture will become reality. In Act 2 there is a meeting of some of the world’s most famous mathematicians. Euler tells the colleagues about Stark’s invention. The Indian mathematician Gupta is very enthusiastic because the so called quantum computer could find out “whether there are any odd perfect numbers and search for new perfect numbers”. The other mathematicians are anxious; they cannot cope with the idea of a super computer. After finding out that the computer has started to prove the Goldbach’s conjecture, they are completely shocked. They decide that “Goldbach must be protected”. In that moment Stark, who was not invited, appears at the meeting. He does not want to accept the argument of the mathematicians that Hal must be turned off and Euler shoots him. In Act 3 we see the final confrontation between Euler and Hal. Euler wants to convince Hal to stop proving the Goldbach’s conjecture. Hal disapproves and Euler turns it off. With the big explosion the computer finishes its “life”.

## **Rainer Heinrich, Germany**

### *Modern Mathematics lessons with Technology and central school-leaving exam? Does this go together?*

There are central written exams (Abitur) in Germany in 15 of 16 federal states meanwhile. The author himself was head of the author group for the written exams in mathematics in Saxony from 1992 to 2002. By a fundamental change of the performances of mathematics lessons to a stronger orientation on the acquisition of competences but also by the availability of new media lessons and central exams have changed the mathematics at school in the last years.

Many countries have introduced the central abitur in Germany within the last 10 years. Furthermore the countries have for Bavaria, Hamburg, Niedersachsen, Mecklenburg, Saxony and Schleswig-Holstein in 2014 introduced a country general central abitur even for the first time.

In connection with this, there are a lot of questions, it was to look for solutions:

What is the role and sense of technology in the lesson and in exams?

How do the pupils have to document solution ways?

How should the structure of an exam be chosen?

Which role plays parts with paper and pencil in the exams?

What is an "easy" task in future?

Which contents should be taken into account in a mathematics lesson with technology?

In the lecture some of these questions shall be illustrated with different examples of abitur tasks.

[Heinrich\\_Englisch.pdf](#)

## **Peter Hofbauer, BHAK/BHAS Horn, Austria**

### *Erfahrungen mit CAS/GTR-SchülerInnen an der Fachhochschule*

Seit vielen Jahren werden parallel zu den Prüfungsergebnissen in den Fächern Wirtschaftsmathematik und Wirtschaftsstatistik an der FH des bfi Wien (Studiengang "Europäische Wirtschaft und Unternehmensführung") aus privaten Interesse auch die "Zubringerschulen" und die von den Studierenden eingesetzte Technologie erhoben. Vor vielen Jahren viel in der Studiengangsleitung und unter den LektorInnen die Entscheidung, jede Technologie in den Lehrveranstaltungen und in der Prüfungssituation zuzulassen. Im Vortrag soll darüber berichtet werden, wie diese Entscheidung die Konzeption der Lehrveranstaltungen und der Prüfungssituation im Laufe der Jahre verändert hat. Insbesondere soll auch auf die durchaus interessanten und vor allem überraschenden Ergebnisse der Auswirkung der Technologie auf die Prüfungsergebnisse berichtet werden. Auch wenn die Erhebung keinerlei allgemeine Rückschlüsse auf studentisches Wissen und Auswirkung von Technologie zulässt, so liefert es doch eine Basis für anregende Diskussionen.

[Hofbauer Erfahrungen.pdf](#)

**Peter Hofbauer, BHAK/BHAS Horn, Austria**

*Wie viel bzw. welches CAS benötigt man für die Zentralmatura  
Angewandte Mathematik an BHS*

Viele LehrerInnen stehen vor der Entscheidung, welche Technologie sie künftig im Mathematikunterricht einsetzen sollen. Für die BHS gibt es eine klare Vorgabe an Mindestanforderungen, die jedes eingesetzte System erfüllen muss. Offen bleiben aber die wesentlichen Fragen: Welche Technologie lässt sich am Besten in den Unterricht integrieren? Welches System "hilft" den Schülerinnen in der Prüfungssituation am meisten? Soll man eher auf GTR oder auf CAS setzen? ...

Im Vortrag wird auf diese Fragen eingegangen und ein kurzer Bericht über den aktuellen Stand der sRDP AM gegeben.

[Hofbauer\\_Wie\\_viel.pdf](#)

**Martin Kazda, Jihočeská univerzita, Czech Republic**

*The Derivation of Kepler's three Laws using Newton's Law of Gravitation  
and the Law of Force*

Nowadays, there are a lot of studies involved in this derivation. These works are primarily based on the academic knowledge of mathematics and physics. Therefore they are suitable for students of technical universities.

My paper offers a simple, straightforward derivation which uses mathematical apparatus and does not introduce additional quantities. That's the reason why it can be used in large extent in secondary schools. In the case of complex mathematical apparatus, as in the derivation of Kepler's Second Law, appropriate graphic mathematical software such as GeoGebra can be used.

It can be also used as an example of interdisciplinary relations between physics and mathematics. This simplified method may prove to pupils and students, what an appropriate mathematical apparatus applicable, while offering teachers and professors possibility to fuse teaching physics and mathematics as an optional mathematical physics seminar.

[Kazda.pdf](#)

**Jiří Kopecký, Cesky Budejovice, Czech Republic**  
*Using computable Document Format in Teaching Mathematics*

Interactive graphics is focused on natural senses and helps students to develop analytical thinking in the way from single perception and experience to general knowledge.

Computable document format (CDF) is a file format from Wolfram Research Company readable in Free CDF Player. This software contains Mathematica's core, providing algorithms for visualization in 2D and 3D and functions for computation on many topics. It allows to insert interactive graphics in teaching materials by authors simply and in a comfortable way. Such textbooks in CDF can contain interactive tools for inquiry based learning with no demand on readers' programming ability.

My research focuses on teaching statistics at university level in an innovative way. In this lecture I will discuss some possibilities and difficulties in using CDF for deploying math textbooks. I will demonstrate mentioned technology in selected exercises from introduction to statistics. The traditional approach to teaching statistics dominates in majority of Czech schools and this subject is unpopular among students.

Our course design gives the opportunity to work intuitively with advanced tools of statistics before their formal introduction.

[Kopecky.pdf](#)

**Lilla Korenova, Bratislava, Slovakia**  
*Computer, Tablet or Graphing Calculator*

Digital technologies have made their way not only into our everyday lives, but nowadays they are also commonly used in schools. Computers, tablets and smartphones are now part of the lives of this new generation of students, so it's only natural that they are used for educational purposes as well. Besides the interactive whiteboards, computers and laptops that are now widespread in schools, students with tablets, smartphones or graphic calculators are not an exceptional sight either. For math teachers it's important to know how and which of these digital technologies can be used during lessons. It seems like that each of the aforementioned technologies has its place and can be effectively used in achieving the objectives of teaching mathematics in secondary schools. In this presentation we would like to point out a few examples on how to use the computers, tablets or graphic calculators effectively and their specifications for various topics, teaching methods and teaching forms. With a few examples we would also like to present the opportunities lying in the application of digital technologies for educational purposes in mathematics.

[Korenova.pdf](#)

## Matija Lokar, Slovenia

### *The Future of eTextbooks*

Electronic versions of textbooks (eTextbooks) have recently become a hot topic in the educational community. The advent of tablet form PCs has further secured eTextbooks as one of the most frequently discussed issues among educators. How can they be compared to printed textbooks? On their web pages one of the leading publishers states: "*An eTextbook is the complete contents of a printed textbook, delivered in electronic form over the Internet. ... An eTextbook has the same content as a printed textbook, the same chapter divisions, and the same page numbering. ...*"

We claim this is not the right way to go. A good eTextbook should be quite different from a printed textbook. What are the main characteristics of a good eTextbook, then? It should provide much more besides the obvious additions and improvements such as interactivity, usage of multimedia, and ease of navigation.

As the need for individual approach towards each student is becoming more and more emphasized, one of the crucial changes we should expect and require is that an eTextbook should allow for customization and personalization. The customization should be achieved on different levels: be it technical (adaptation to different output devices where the eTextbook is to be used, tools used in examples and so on) or (more importantly) be it the adaptation of its content.

But where should this customization occur? Should the publishers provide all the possible variations? Should the learners decide for themselves what is the most appropriate for them? We think that customization will be one of the most important parts of a teacher's duties in the future. It is the teacher who is capable of making the proper combination of the teaching resources available, making it suitable for the particular pedagogical situation. Who if not the teacher is the most capable of assessing what is the preferred learning style of a specific learner, or which approach works best for each of his students. The teacher knows what tools are available, which technological skills his students possess ... The personal characteristics of each teacher should not be neglected either; namely the way of teaching, personal beliefs, opinions regarding motivational approaches, etc. Therefore eTextbooks should be designed to be adaptable to the pedagogical situation and to the user, be it a learner or a teacher.

Consequently, eTextbooks should be quite different from the printed editions. It should be a given that an eTextbook allows and enables uncomplicated customization and personalization. The talk will show some actual examples where some of the proposed ideas have already been implemented as well as a description of more practical ways showing what other concepts should look like.

**Jeanett López Garcia, Jorge J. Jiménez Zamudio, E. Beatriz Clavel Diaz, FES Acatlán-UNAM, México**

*From Calculus to Dynamical Systems through CAS*

Mathematics environments such as GeoGebra or Maple or Web resources, some of them mixed with MATLAB, can be used to improve the understanding of many mathematical topics, such as osculating plane attached to a particle journeying on 3D curve, limit cycle in Dynamic System, and even more basic topic as discover the behavior of the primitive function from the own derivate graphic. We will show a selection of this kind of applications in the classroom.

But not only CAS could be useful for achieving mathematical performance of several functions and its dynamic behavior. We, also, will show how a simple spreadsheet allows us to get the bifurcations diagram for the logistic function.

Keywords: Mathematical animations, simulating dynamic processes, CAS.

[Lopez Garcia.pdf](#)

**Miroslav Lovric, McMaster University, Canada**

*Math with Programming – Shaken or Stirred?*

In this presentation I will report on the classroom experience (from both students' and instructor's perspectives) in an advanced mathematics problem-solving course (called Math 3G3), in which students were asked to investigate mathematical problems using computers. In this case, 'using computers' means writing one's own computer code, as opposed to using ready-made software where users manipulate parameters, change settings, etc., but are not (in most cases) aware of underlying mechanisms that control the software. Students were given freedom in choosing a programming language (most common choices were Maple, MATLAB, Python and C++). However, in order for all of us to communicate, I asked students to use a pseudocode, for instance on assignments and tests. Even though students in Math 3G3 were good mathematics students (all majors in mathematics), programming turned out to be quite challenging. I will comment on the conflicts between my expectations and my students' experiences, and on a number of cognitive situations that emerged, all of which will help me adjust the course the next time I will teach it. Since Math 3G3 included paper and pencil problem-solving activities as well, I was able to contrast students' thinking, approaches to solving problems and attitudes in using two distinct media (paper and computer), and came up with interesting and useful insights. I will speak about the tension between the paper and the computer as problem-solving tools, and offer suggestions for a possibly more smooth transition between the two media. Although the context of Math 3G3 is upper-year university mathematics, I believe that the observations and messages are relevant for almost any programming-intensive mathematics course in a tertiary institution.

[Lovric.pdf](#)

**Eugenia Marmolejo Rivas, Universidad Nacional Autónoma de México**  
*Modelling the Motion of an Elevator*

In this lecture possible models for the motion of an elevator are explored. We will discuss the following three tasks:

1. Students are given a model that represents the position of the elevator. They are asked to evaluate its usefulness and identify the problems of the given model so that they can create a better model.
2. Students are asked to model the movement of the elevator. No initial model or data are given.
3. Students take a video of the movement of the elevator and use Tracker video analysis and GeoGebra to model the motion of the elevator.

We discuss student's answers to find out if technology is used in a significant way that enhances the development of the task.

[Marmolejo.pdf](#)

**Sirje Pihlap, Estonia**  
*Opinions of Teachers on the Use of the WIRIS Computer Algebra System for Teaching and Learning Mathematics*

The previous academic year was the third year of implementation of the new curriculum in Estonia. An important element of the curriculum is the use of mathematics software – it is now mandatory and the curriculum also specifies some learning outcomes to be achieved using mathematics software. The requirements of the curriculum can be met, to a large extent, through three software systems: T-algebra for step-by-step solving of algebra problems, Geogebra dynamic geometry software, and Wiris computer algebra system. A number of training courses on these three software systems have been organised for teachers. GeoGebra has been the most popular application with 63% of mathematics teachers having used it for teaching, while the Wiris system has only been used by 35 % of mathematics teachers (Prei, 2013). This study analyses the opinions of the 85 teachers, who participated in Wiris training, on the use of this system in mathematics education. The analysis also covers teachers' opinions on some distinctive traits of Wiris. Like any computer application, Wiris has certain distinctive features that could prove to be either obstacles to learning or useful teaching tools.

[Pihlap.pdf](#)

**Karsten Schmidt, Schmalkalden University of Applied Sciences, Germany**  
*Motivating Students in an Introductory Matrix Algebra Course*

Linear algebra is certainly an area of mathematics of increasing importance. Unfortunately, most business and economics students do not appreciate courses in mathematics and statistics. This is particularly true for the (introductory) matrix algebra course because most topics are rather abstract and easy-to-apply examples are difficult to find.

However, magic squares can be very helpful in stimulating students' interest in matrix algebra and are easy to apply. A magic square of order  $n$  is a square arrangement of  $n^2$  real numbers, such that the sum of the elements in each row, column, and diagonal is equal to a constant  $s$ , its magic sum.

Many interesting activities can be carried out in class at very different stages of the course, using a Computer Algebra System to facilitate computations.

The introductory matrix algebra course takes place every summer semester. This year, magic squares will be used for the first time throughout the course. The idea of using magic squares as examples arose one year ago, but in the 2013 course, the last lecture consisted of a repetition of many different topics using magic squares, and students clearly appreciated this.

Any  $3 \times 3$  magic square  $M$  can be written as the sum of two matrices,  $M = sG + N$ , where  $G = 1/3J$  ( $J$  denotes the matrix of ones), and also  $N$  has a simple structure defined by only two real numbers. This allows additional interesting activities.

Magic squares from different times and regions, like the ancient Chinese  $3 \times 3$  Lo-Shu (4,9,2;3,5,7;8,1,6), will be used as examples in the presentation.

Schmidt\_Motivating.pdf

**Alla Stolyarevska, The Eastern-Ukrainian Branch of the International Solomon University, Yuriy Kuznyetsov, Research Production Enterprise Hartron-Arkos, Natalia Shunina, Central Clinical Hospital of Ukraine Rail, all Kharkov, Ukraine**  
*Using a Mathematical Package for Modeling of Sleep Disorders in Patients with Traumatic Brain Injury*

Mathematical packages are widely used for data processing and analysis, including processing of biomedical information. In this paper we consider the use of known mathematical packages for processing of biomedical data on patients who received a traumatic brain injury in order to determine the effect of injury severity on sleep. The importance of the problem is related primarily to the fact that persons, who have had a traumatic brain injury, also have and sleep disturbances. As a consequence, the quality of life and ability to work are declined. Traumatic brain injuries include both medical and psychiatric symptoms, and frequent complaints of sleep disturbance. Special studies are required to determine the nature and extent of this disturbance. The researchers need to focus on the disclosure of specific types, causes and severity of traumatic brain injury, which often lead to problems of sleep disorders. The researcher builds its own “model of sleep disorders”.

The models allow uncovering the hidden relationships between measures of sleep disorders and their significance due to the severity of injury.

Algorithm, built on the basis of models, makes it possible a certain degree of confidence to identify the severity of traumatic brain injury patients, thereby determine the clinical determinants of quality of life in patients who have suffered traumatic brain injury with sleep disorders.

We used mainly R-package and Excel. R-package is an open source free software. It includes a lot of libraries which are created by users in different countries. I will compare the results obtained with using Statistica and Maple.

Keywords: Mathematical packages, data analysis, traumatic brain injury, sleep disorders, quality of life.

[Stolyarevska.pdf](#)

**Irena Štrausová, Czech Republic**  
*Dynamic visual proofs in mathematics education*

Pictures and diagrams play an important role in the process of understanding various mathematical features. Moreover, an appropriate picture or diagram can be used as a visual proof of some quality or theorem. These non-verbal proofs are more attractive and acceptable to students than the classical proofs. Their educational potential is no doubt high, but compared to classical proofs, their weak point is in the fact that they mostly do not capture the chain of thought leading to the proof but only the result. This deficiency, which arises especially in the educational process, can be simply removed by the use of dynamic software – e.g. dynamic geometry or algebraic software. The paper will present selected examples of such dynamic visual proofs in the form of materials that we use in the teaching of secondary school mathematics and in the mathematics teacher training programs.

Some of mathematics teachers were asked to use dynamic visual proofs during their lessons. The paper also provides samples of video studies from these lessons and some insights from these video studies and interviews with teachers.

[Strausova.pdf](#)

**Eno Tõnisson, University of Tartu, Estonia**

*Differences between Expected Answers and the Answers Given by  
Computer Algebra Systems to School Equations*

Equation solving is an important part of school mathematics curricula. Equations are also important in the use of Computer Algebra Systems (CAS). Sometimes a CAS offers an answer that is somewhat different from the answer that is probably expected by the student or teacher. (The customs and requirements for answers can also differ between regions or even teachers.) The expected answer and the answer offered by a CAS could be equivalent or non-equivalent. The gap between the answers could be bridgeable in an instructive manner. These (somewhat unexpected) answers could serve as a catalyst for rich mathematical discussion (see (Pierce and Stacey, 2010)).

In this study, over 60 equations (linear, quadratic and fractional equations, equations that contain an absolute value of an expression, irrational, exponential, logarithmic, trigonometric and literal equations) were solved using different CAS. Many answers offered by the CAS coincide with the expected answers. This paper focuses on the answers that deviated from expectations in some form. We considered the following questions in case of each of those answers: Is the answer offered by a CAS equivalent to the answer required at school? Would it be easy for a student, working manually, to transform the answer into the required form? How about if the student uses a CAS? In this context it is important to analyze potential instructiveness such situations. What issues could be raised for discussion? Some of them seem to be relevant in a school context: for example, the properties of different functions (like square root, logarithmic or trigonometric functions). Some issues, e.g., pertaining to infinity, could be beyond the regular school level.

The aim is to present a spectrum of possible differences and to highlight the situations (equations, CAS) in which a particular type of discrepancy could occur.

[tonisson\\_Equation.pdf](#)

**Eno Tõnisson, Marina Lepp, University of Tartu, Estonia**

*Students' Comparison of Their Trigonometric Answers with the Answers of a Computer Algebra System in Terms of Equivalence and Correctness*

The answer offered by computer algebra systems (CAS) can sometimes differ from those expected by the students or teachers. Such answers could serve as a catalyst for rich mathematical discussion (see (Pierce and Stacey, 2010)). An important topic is the equivalence/non-equivalence of the students' answers with the CAS answers. It is also relevant in case of equations and could be particularly helpful when working with trigonometric equations, which can often have quite sophisticated answers. Are students able to ascertain equivalence/non-equivalence? How do they understand correctness of the answers? Are there any differences in this regard between different types of equations and answers?

The paper is based on the lessons of a course in elementary mathematics to first-year students who had very limited experience with CAS. The students worked in pairs and their discussions were audio-recorded. They had worksheets with trigonometric equations and questions. The equations to be solved were in a prescribed order. For each equation there was a particular CAS that gave a different answer from the expected answer of the students.

Initially, the students solved an equation (correctly or not) without a CAS. Then they solved the same equation with a particular CAS. They were given questions, guiding them to analyze the differences, equivalence and correctness of their own answers and CAS answers.

Ca 200 instances of equation-solving by ca 50 pairs of students were analyzed. The data consists of the students' worksheets and audio records of their discussions. The worksheets provide information on equivalence/non-equivalence of the students' and CAS answers. It is determinable through mathematical reasoning by researchers. The second dimension is the students' opinion about the equivalence/non-equivalence. That is based on an analysis of the worksheets and audio data. Different types of equations can be associated with different obstacles to clarification of equivalence/non-equivalence. For example, there seems to be quite a high degree of confusion about the meaning of  $n$  in the answers to trigonometric equations, even if it is used correctly in solutions.

It seems that comparing their answers with CAS answers was an interesting task for the students. In most cases this led to active discussions on trigonometry. The productivity of the discussions is a subject matter for another study but some points are also briefly mentioned in this paper.

[tonissonlepp\\_Students.pdf](#)

**Melih Turgut, Candas Uygan, Eskisehir Osmangazi University, Turkey**  
*Designing Spatial Visualization Tasks for Middle School Students with a  
3D Modelling Software*

Spatial ability can be defined as the abilities of imagine the visualization of an object from different viewpoints, rotation of it and blend or integrate of the parts of the given object (McGee, 1979; Olkun, 2003; Turgut, 2007). An arbitrary person in daily life uses this ability, for instance, while walking in a street considering his/her actual position in the city or placing the wares in a room. McGee (1979) proposed two components of spatial ability: *spatial visualization* and *spatial orientation*. Spatial visualization was defined as “the ability to represent and manipulate visual objects mentally” (Turgut & Uygan, 2013, p. 292). Spatial orientation can be defined as the ability of visualize views of given objects from different viewpoints.

Spatial thinking has an important role in the teaching and learning of mathematics process. And studies showed that this ability has positive correlations with geometry and mathematics education. Besides, recently, some of mathematics questions that required the use of spatial thinking have been appeared in national exams of Turkey (Uygan & Turgut, 2012). In this respect, considering its important role in mathematics education, development (training) of spatial ability by the aid of information and communication technologies (ICT) had great attention in the reviewed literature (Kurtulus, 2013; Turgut, 2010; Turgut & Uygan, 2013; Turgut & Uygan, 2014). With similar aim, this study describes the importance of spatial thinking and thereafter, by the aid of a 3D modelling software SketchUp®, we designed some special activities in order to develop middle school students’ spatial visualization skills. In the design of the activities, two sub skill of the spatial visualization; *mental rotation* and *mental integration* are considered. Two theoretical frameworks; Duval’s (1998) *cognitive process of geometric reasoning* and *instrumental integration* (Assude, 2007; Verillon & Rabardel, 1995) are based. By the following instrumental integration approach, first, SketchUp’s special tools (pan, rotate, protractor, orbit etc.) will be introduced (*instrumental initiation process*) and 6 activities will be given to students in a computerized environment. The tasks have been designed with 3 figures in a SketchUp interface; each is made of a square. In the task, it is asked from students how first two squares should be manipulated in order to form the third figure. One sample task is given in the Figure 1.

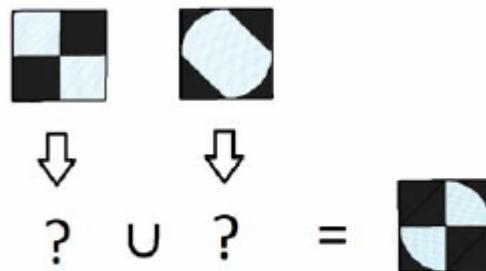


Figure 1: *Sample task*

Each task has two main steps; the first is mental rotation and the second is mental integration. These steps were inspired from Lee (2005). So students have to use protractor with a determined angle and move the squares to form the third figure. Those activities will be piloted in a computerized environment in order to investigate students reasoning process with respect to Duval’s (1998) framework. The results will be presented in the conference.

**Ilyas Yavuz, Ibrahim Kepceoglu, Abdulkadir Kerpic, Marmara University, Turkey**  
*Question Types for Assessment of Mathematics Education in Dynamic Geometry Environments*

Assessment of student learning requires the use of techniques for measuring student achievement. Assessment is more than a collection of techniques, however. It is a systematic process that plays a significant role in effective teaching. It begins with the identification of learning goals, monitors the progress students make toward those goals, and ends with a judgment concerning the extent to which those goals have been attained. In the literature, the integration of technological tools into the assessment process is expressed by the “e-assessment” concept. Assessment using ICT has come to be known as e-assessment, which includes the entire assessment process, from designing assignments to storing the results with the help of ICT. The most of the researches related to the e-assessment focused on how to transfer the examinations in paper-pencil environment to the electronic environment. In addition to those studies, in the research about the integration of computer assisted assessment tools to the mathematics instruction, the participant pre-service teachers use these tools in favor of the formative assessment. In a similar study, pre-service teachers prepared lessons and home works using technological tools such as dynamic geometry software and thus it is shown that those tools can be used in the process of assessment. In the literature, there is numerous studies concerning technology supported mathematics instruction. But none of them is related to the assessment dimension of the mathematics teaching. In particular, it is highly recommended to conduct researches about the use of technology in that dimension. The aim of this study is to determine question types that can be used for assessment of computer based mathematics instruction. Case study is considered as the methodology of the research. The reason is that a case study investigates a contemporary phenomenon in depth and within its real-life context when the boundaries between phenomenon and context are not clearly evident. 15 pre-service mathematics teachers have participated to the study of 6 weeks (3 hours per week). Every pre-service teacher worked individually on totally 26 geometry questions of different types. At the same time, they explained their approaches to solve problems by filling out a paper. In addition, a screen capture program is used to grab the video of what is seen on the computer screen. Meanwhile the researchers observed pre-service teachers’ work on computers. The findings of the study are revealed via the analysis of each video, of their comparison with the written explanations and of the researchers’ observations. Based on the findings, the most significant result of the study is that open-ended geometry problems reflect students’ knowledge better than other types of questions. Moreover, all types of questions should be used in order to assess students’ learning performance.

**Walter Wegscheider, PH NÖ, Austria**  
*Elektronische Prüfungsumgebungen*

Mit dem Begriff einer elektronischen Prüfungsumgebung werden im Schulbereich zwei durchaus divergente Ansätze angesprochen. Zumeist ist der Begriff verknüpft mit einer „sicheren“ Umgebung, in der Prüfungen beim Einsatz von Technologie (an einem PC in einem EDV-Saal der Schule, am Notebook oder bei Verwendung anderer technischer Hilfsmittel wie zB. algebratauglichen Taschenrechnern) abgehalten werden können, ohne dass der unerlaubte Zugriff auf vorher abgespeichertes Material auf diesen Geräten bzw. unerwünschte Kommunikation innerhalb der Klasse oder auch nach außen möglich ist. Solche Systeme sind in verschiedener Form und meist standortspezifisch adaptiert an Schulen teilweise seit über 20 Jahren im Einsatz.

Im Zusammenhang mit zentralen Abschlussprüfungen wird der Begriff der elektronischen Prüfungsumgebung immer öfter auch als Synonym für spezielle Prüfungstools genannt. Diese sind Komplettsysteme, die von der Aufgabenstellung über die Lösung derselben mit möglichen Hilfsmitteln bis zur Abgabe der Prüfung und schließlich oft auch Hilfsmittel zur Verbesserung integriert bereitstellen.

Verschiedene in Österreich verwendete Systeme werden vorgestellt, ihre technische Realisierbarkeit und ihre Auswirkungen auf die Art der Prüfung (Fragen) besprochen.

**Sharma Jyoti, University of Delhi, India**  
*Changing the Course of Mathematics Education: A Case Study from India*

Mathematics Education is an area of concern in India as many students do not find mathematics interesting whereas many develop a psychological phobia towards the subject. Many students drop mathematics after grade X as mathematics is optional after grade X. Adults who had left mathematics after grade X continue to live with the same phobia for mathematics. Though India has come up as a leader in technology at the world stage but the influence of technology has not reached to the grass root level within the country. Challenges of education in the country can be minimized by optimizing the use of technology. The present paper presents a unique experiment initiated by the two premium institutes of higher education in India. It shares the dynamics of first meta university based course, “Master of mathematics Education” program started in India.. The immediate aim of the course is to prepare professionally inclined mathematics teachers who have sound knowledge of subject matter, who are specialized in constructive and alternate mathematics pedagogy and comfortable in the use of technology in mathematics teaching. The larger aim of the course is to develop mathematics education as an academic discipline to foster research and innovation in multiple aspects of mathematics teaching and mathematics as a subject. The present paper highlights the unique features of the course and how it aims to revolutionize mathematics education in the country.

# Workshops

**Wolfgang Alvermann, BBS II Emden, Germany**  
*Technical Problems – solved by Sec 2 Mathematics*

In central examination of vocational schools in Lower Saxony real life problems from the field of technology or economy using a GTR or CAS are obligatory. In text books you will find only few suitable problems so the teacher has to develop problems by himself.

Problems from industrial companies and crafts enterprises provided by students form the base of this workshop in which is shown that applying Secondary 2 level mathematics enables to find satisfying solutions for these problems under integration of a CAS.

Contents of the workshop is:

- DOT PRODUCT AND VECTOR PRODUCT (production of a special frustum of a pyramid),
- LENGTH CALCULATION WITH INTEGRAL CALCULUS (Cut of cylinders in metal processing),
- REGRESSION AS AN EXTREMAL PROBLEM (Bending problem in connection with special tests for welds of submarines),
- FRICTIONAL PROBLEMS which lead on TRIGONOMETRIC- and EXPONENTIAL FUNCTIONS (Eytelwein's equation for rope friction),
- DEFLECTION CURVES, represented by polynomial functions.

Special technical and physical knowledge is not required. This workshop course is suitable for teachers of the vocational schools and general secondary schools as well. Basic knowledge in dealing with a CAS is recommended.

Technology: TI-NspireCAS

Participants with basic knowledge in other CAS are very welcome.

Vorgabe beim Zentralabitur an den niedersächsischen Beruflichen Gymnasien sind berufsbezogene Aufgaben aus den Bereichen Technik oder Wirtschaft unter Einbeziehung eines GTR bzw. CAS. Wirkliche Anwendungsprobleme sind in den Schulbüchern nur ansatzweise zu finden, eigene Aufgaben zu entwickeln bleibt daher Aufgabe des Lehrers.

Probleme aus Industrie- und Handwerksbetrieben, von Schülern in den Unterricht eingebracht sowie selbst entwickelte Aufgaben bilden die Grundlagen dieses Workshops, in dem gezeigt wird, dass die Schulmathematik der SEK II unter Einbindung eines CAS geeignet ist, für diese Aufgaben Lösungen zu finden. Inhaltliche Probleme dieser Praxisaufgaben sind

- SKALAR- UND VEKTORPRODUKT bei der Herstellung eines stumpfen Körpers für eine Verzinkerei,
- LÄNGENBERECHNUNG MIT INTEGRALRECHNUNG für einen Schnittkörper in der Blechverarbeitung,
- REGRESSION ALS EXTREMALPROBLEM bei einem Biegeproblem in der Werkstoffprüfung,
- REIBUNGSPROBLEME, die auf TRIGONOMETRISCHE- und EXPONENZIALFUNKTIONEN führen, sowie
- BIEGELINIEN, die durch ganzrationale Funktionen dargestellt werden können

Besondere technische und physikalische Kenntnisse werden nicht benötigt, so dass der Kurs sowohl für Lehrkräfte der berufsbildenden Schulen als auch der Gymnasien geeignet ist. Kenntnisse im Umgang mit einem CAS sollten vorhanden sein.

Technologie: TI-NspireCAS

[Alvermann\\_Deutsch.pdf](#)  
[Alvermann\\_Englisch.pdf](#)  
[Alvermann\\_WS.tns](#)

### **Tim Bebensee, CASIO**

*Workshop für CASIO Class Pad II*

Dieser Workshop bietet die Möglichkeit den modernen CAS-Rechner ClassPad II kennenzulernen und in ausgewählten kompetenzorientierten Beispielen zu erproben. Durch die innovative Stift-Bedienung fällt die Einarbeitung in der Regel leicht, so dass am Ende des Workshops auch eigene Beispiele erprobt werden können, wenn gewünscht. ClassPad II-Geräte werden gestellt.

[Bebensee\\_WS.pdf](#)  
[Bebensee\\_Appendix.pdf](#)

### **Joseph Abi-Nader, Maplesoft**

*From Exploration to Assessment using Maple T.A.*

During this workshop, you will have the opportunity to learn how Maple T.A., an automated testing and assessment system designed especially for courses involving mathematics, works from both the an instructor and a student view point. We will demonstrate common operations, such as creating and taking quizzes and providing feedback, and explore the possibilities offered by more advanced options such as adaptive assignments and flexible grading.

By attending this workshop, you will acquire an understanding of the latest developments in distant and classroom e-testing and e-assessment, and how these systems empower teachers to enhance and assess their students reasoning and problem-solving skills.

## **Gertrud Aumayr, KPH Wien/Krems, Austria**

### *„Verstehensorientiert“ Unterrichten mit Blick auf die neue Reifeprüfung*

„Aufgaben zum Lernen“ sollten sich deutlich von „Aufgaben zum Leisten“ unterscheiden. Während bei „Aufgaben zum Lernen“ Motivation durch spannende Aufgaben, explorativer Umgang mit eigenen Fehlern, Entdecken von eigenen Lösungswegen, Kooperation und Kommunikation im Fokus stehen können, orientieren sich „Aufgaben zum Leisten“ auf Fehlervermeidung, Einzelleistung und Auswertbarkeit. Die Reaktion auf die neue Reifeprüfung kann daher nicht ein reines Training von Multiple Choice Aufgaben sein.

Technologie spielt bei den „Aufgaben zum Lernen“ eine hervorzuhebende Rolle. Schüler nützen dieses Werkzeug zum Experimentieren, d. h. sie variieren gezielt Größen und beobachten die Auswirkungen ihres Handelns, zum Exaktifizieren und zum Anwenden. Im Workshop werden spannende „Aufgaben zum Lernen“ bearbeitet und Verbindungen zu bereits veröffentlichten Musteraufgaben der zentralen Reifeprüfung hergestellt. Für die Teilnehmer stehen dafür iPads mit TI-Nspire App zur Verfügung.

Der Workshop wird mit TI-Nspire auf Leih-iPads durchgeführt.

[Aumayr.pdf](#)

## **Josef Böhm, ACDCA & DUG, Austria**

### *GINI-Coefficient and GOZINTO-Graph*

GINI-Coefficient together with LORENZ-curve and GOZINTO-Graphs are economic applications of secondary school mathematics. They are part of the curriculum in Austria's Colleges for Business Administration (= Handelsakademie).

The GINI-Coefficient is a means for measuring the income distribution – and other distributions - of a nation's population. It is commonly used as a measure of inequality of income or wealth.

This sounds a little bit complicated but in fact it is an easy to perform application of integration based on the modelling of the so called LORENZ-Curve. We should be happy to find an application of definite integral which is not only calculation of the area between two curves.

The GOZINTO-Graph or GOZINTO-Chart is a pictorial representation that shows how the elements required to build a product fit together. This is often the first step for several planning functions especially for material requirements planning, for time and manpower resources planning, for production cost, etc.

This chart gives the input for building the production matrix. Easy to understand and supported by technology easy to perform matrix calculations are leading to the results.

Students can easily be inspired to design own tasks. Examples of students' works will be presented.

Technology: TI-Nspire, DERIVE or any other CAS

[Boehm\\_GINI.pdf](#)

**Josef Böhm, ACDCA & DUG, Austria**  
*Solving Brain Teasers/Twisters – CAS Assisted*

Most of brain teasers can be faced using Mathematical Logic procedures. Therefore, using a CAS, which handles with Logic expressions, may help solving this kind of problems. The German weekly newspaper *Die Zeit* offers every week another brain teaser. Just for fun I tried to solve one or the other problem assisted by CAS. We will apply equations, Boolean expressions and other tools in order to solve the problems. We will also try to produce own brain teasers. As this is not an IT-workshop no programming skills are necessary. The tools will be provided. We will focus on the “mathematization process” of the problems. The software chosen for developing this workshop is our old and good friend DERIVE, the core of so many TIME’s previous Conferences. If possible we will use the TI-NspireCAS, too. The procedures presented can be transferred to all other CAS, of course.

[Boehm\\_Twisters.pdf](#)

**Peter Flynn, Australia**  
*CAS and Secondary School Mathematics: Mathematical Tasks in Principle become Tasks in Practice*

The availability and versatility of contemporary technology such as CAS afford upper-secondary school mathematics teachers with opportunities to widen the bandwidth of mathematical abilities and knowledge required of students.

In essence, the amalgamated numerical, graphical and symbolic functionality of a CAS, coupled with dynamic geometry and digital image capabilities convert mathematical tasks formerly deemed as tasks that could only be set in principle to tasks that now can be set in practice.

In this workshop one such CAS, namely TI-Nspire CAS, will be used to analyse a small set of mathematical tasks from this 'in principle to in practice' lens. These mathematical tasks, including Amazing Systems of Equations (covering algebra, matrices, functions and graphs and the Fibonacci sequence) and Anscombe's Quartet (covering univariate and bivariate statistical analysis) will be discussed from the perspectives of achieving curricular goals and effects on teacher practice.

Some basic familiarity with TI-Nspire CAS will be assumed.

[Flynn.pdf](#)

**Guido Herweyers, KU Leuven Kulab, T3 Flanders, Belgium**  
**Koen Stulens, T3 Europe, Belgium**  
*Exploring Mathematics through Multiple Representations*

Technology provides the ability to combine graphic, geometric, numerical and symbolic representations of a mathematical concept. These dynamically linked representations help students to see connections and build understanding, they are teaching and learning tools to get a better understanding of mathematical concepts.

The educational and didactical value, exploring multiple representations of a mathematical problem or concept, will be shown through concrete examples at secondary school level for linear algebra, calculus and statistics.

During the workshop, the examples will be explored using the document based TI-Nspire CX CAS handheld Technology, without assuming a prior knowledge of this technology. It gives students the possibility to save their reasoning when trying to solve a problem or to build a mathematical model, and to reflect back to the problem.

[Herweyers.pdf](#)  
[Herweyers\\_tns.zip](#)

**René Hugelshofer, Switzerland**  
*Teaching Aids for CAS-compliant Lessons*

With the third booklet on "Growth functions" the three authors B. Frei, R. Märki, R. Hugelshofer have completed the teaching materials for Algebra for high schools (previously published "Linear and Quadratic Functions", and "Analysis" from R. Märki).

In the workshop, the teaching materials for Algebra are briefly introduced, and then shown together on the basis of examples and exercises from the four booklets how CAS can reasonably be used as an aid in teaching.

[Hugelshofer.pdf](#)  
[Hugelshofer\\_files.pdf](#)

**Josef Lechner, Austria**  
*Wachstumsmodelle – diskret und kontinuierlich*

Im Workshop soll gezeigt werden, wie mit der mathematischen Arbeitsumgebung GeoGebra eine umfassende und aspektreiche Behandlung von Wachstumsmodellen möglich wird: Schrittweises Nachvollziehen von Entwicklungen in der Tabellenkalkulation, numerischen Berechnungsmöglichkeiten, exaktes Lösen von Gleichungen und Differentialgleichungen mit dem Computeralgebra-System, Untersuchung des Langzeitverhaltens von Systemen sowie vielfältige graphische Darstellungsformen in mehreren Graphikfenstern. Damit ergeben sich neue Möglichkeiten bei der Modellierung von Wachstumsprozessen im Unterricht.

[Lechner Wachstumsmodelle.pdf](#)

**Andreas Lindner, PH OÖ, Austria**  
*Workshop GeoGebra*

Eine Stärke von GeoGebra ist die dynamische Verknüpfung von Geometrie, algebraischer Darstellung, CAS und Tabellen. Anhand von ausgewählten Beispielen aus der Sekundarstufe II (Newton-Näherungsverfahren, Extremwertaufgaben, ...) soll aufgezeigt werden, wie die verschiedenen Module miteinander verknüpft werden können und dadurch Technologie gewinnbringend im Unterricht eingesetzt werden kann. Grundlegende Kenntnisse in der Bedienung von GeoGebra sind erwünscht.

**Robert Märki, T3-Switzerland**  
*Parametric 3D-Plot, ein zeitgemäßes Werkzeug für die Raumgeometrie*

Die anschauliche Geometrie im Raum steht in der Schule im Schatten analytischer Methoden (Vektorgeometrie) wo oft viele Aufgaben zu finden sind, die auf Grund von erlernten Rechenregeln rein syntaktisch und ohne Rekurs auf die räumliche Vorstellung gelöst werden können. Die Schulung und Entwicklung des räumlichen Vorstellungsvermögens wird damit nicht genügend gefördert. Für die anschauliche und konstruktive Geometrie gibt es heute nebst Zirkel und Lineal andere mächtige und einfach zu bedienende Werkzeuge, mit denen wir den Raum sowie räumliche Beziehungen und Formen erfassen beschreiben und darstellen können. Es gilt, diese Werkzeuge auch im Schulbetrieb einzusetzen und ihren Gebrauch zu üben. Die 3D-Graphik des TI-Nspire ermöglicht auf vielfältige Weise Themen der Raumgeometrie zu behandeln und zu erforschen, die bislang in der Schule kaum Beachtung fanden und die in hervorragender Weise geeignet sind, das räumliche Vorstellungsvermögen zu schulen. Es werden Beispiele aus verschiedenen Bereichen gezeigt wie Yachtbau, Pasta-Formen, Formen aus der Tierwelt.

Insbesondere erweisen sich die Schieberegler als besonders hilfreich zum Anpassen der Parameterwerte sowie zum Modellieren von Formveränderungen und einfachen Bewegungen. Die kreative, gestalterische und ästhetische Seite der Mathematik bekommt so ein viel größeres Gewicht, was auch sehr motivierend wirkt. Darüber hinaus können viele dieser Themen für größere oder kleinere Projektarbeiten verwendet werden.

Die Teilnehmenden sollten einen Laptop oder ein iPad mit der TI-Nspire-CAS Software mitnehmen. Handheld werden zur Verfügung gestellt. Es steht wahrscheinlich ein ComputerLab zur Verfügung.

[Maerki\\_Parametric.pdf](#)

**Heidi Metzger-Schuhäcker, Peter Hofbauer, BHAK/BHAS Horn, Austria**  
*Wirtschaftliche Anwendungen in der Schulmathematik*

Nicht nur in den berufsbildenden sondern auch in den allgemeinbildenden höheren Schulen ist anwendungsorientierte Mathematik von zunehmender Bedeutung. Ziel des Workshops ist die Vorstellung und gemeinsame Erarbeitung praxisnahen Unterrichtsmaterials für den Mathematikunterricht mit ökonomischen Anwendungsbeispielen, um Interesse für wirtschaftliche Zusammenhänge zu wecken und den anwendungsbezogenen Mathematikunterricht praxisnäher zu machen. Dabei werden Beispiele aus der Finanzmathematik zum Thema Rentenrechnung und kontinuierliche Zahlungsströme vorgestellt, aber auch Aufgaben aus der Kosten- und Preistheorie präsentiert, die Gewinnmaximierung, lang- beziehungsweise kurzfristige Preisuntergrenzen, Elastizitäten, Konsum- und Produzentenrente oder auch Preisindizes beinhalten. Nicht zuletzt soll der Einsatz von Computeralgebrasystemen speziell bei solchen ökonomischen Anwendungsaufgaben und sein Mehrwert in der Schulmathematik gezeigt werden.

[Metzger.pdf](#)

**Thomas Müller, Austria**

*Analytische Geometrie im Raum – visualisiert und technologiegestützt mit TI-Nspire*

Der Workshop widmet sich der Umsetzung des Inhaltsbereiches Algebra und Geometrie der Sekundarstufe 2, wie er für die zentrale standardisierte schriftliche Reifeprüfung in Mathematik in Österreich vorgesehen ist. Speziell geht es um das Arbeiten mit analytischer Geometrie im Raum. Neben der Interpretation von Geradengleichungen und Lagebeziehungen zwischen Geraden und zwischen Punkten und Geraden soll das Eingehen auf die Förderung der Raumvorstellung nicht zu kurz kommen. Die Technologie Nspire bietet dazu ein hervorragendes Visualisierungstool an, um besonders bei raumvorstellungsschwachen Lernenden die Möglichkeit einer konkreten Verbindung zwischen algebraischen Gleichungen und dreidimensionalem Raum herzustellen. Im Workshop wird mit der Softwareversion auf den eigenen Laptops gearbeitet und der 3D-Raum visualisiert.

[Mueller.pdf](#)

## **Kurt Söser, BHAK Steyr, Austria**

### *Interactive Mathematics-Workbook on a pen-enabled display*

Since five years I am using a pen-enabled tablet-PC in my maths teaching profession on an everyday basis at the business college in Steyr. In my opinion the stylus input is essential for every mathematician (formulas, sketches,...) but can be far more than just a pen. These pen-enabled devices are getting cheaper and cheaper and have reached the price tag of a CAS-Calculator. But with the power of a fully grown PC and all its possibilities there are many educational scenarios in the classroom.

In this workshop I want to show three things:

- I want to show how I use GeoGebra and OneNote and other tools in my everyday teaching and give some examples on how I use these tools to get 21st century mathematics into the hands of students.
- I am going to demonstrate the use of GeoGebraBooks, a free online platform for creating a collection of GeoGebra applets and every workshop attendee can explore the possibilities of GeoGebraTube
- In addition to that I will show the interactive Word 2013 Plugin of GeoGebra and its use for small learning textbooks.

To contribute to the ongoing discussion about the "new" format of textbooks, I am developing a kind of interactive, multimedia workbook (CC-BY-SA) on the basis of Microsoft OneNote (free since March 2014) with lots of GeoGebra-Applets (from geogebraTube.org), videos and other learning resources in it. The aim is to get a free, reusable and personal workbook for statistics and probability in the first place (more to follow). This workbook can easily be adapted and personalized by every teacher and every student to make it their "own". It should be more than just a digital textbook. It should provide the teacher AND the student with lots of "digital" content for a blended learning model and give them the power of the stylus input to make personal annotations. In the workshop I want to give a short demonstration of the work in progress, start a discussion about my idea and get inspiring ideas from the attendees.

[Soeser.pdf](#)

## **Christian Zöpfl, HTL Braunau, Austria**

### *Interaktivität & Technologieeinsatz im Unterricht*

Ein zentraler Punkt des Mathematikunterrichts sollte die Interaktion zwischen Schülerinnen & Schülern und der Lehrperson sein. Oft stellt gerade der Einsatz moderner technologischer Hilfsmittel wie Taschenrechner oder Laptops die Lehrperson hier vor nicht zu unterschätzende Hürden. Echte Interaktion zwischen Lernenden und Lehrenden wird aufgrund der technischen Rahmenbedingungen durch die schlichte Präsentation vorgefertigter Inhalte durch den Lehrenden ersetzt.

In diesem Workshop soll der TI Navigator als ein mögliches Hilfsmittel bei einer interaktiven Gestaltung des technologiebasierten Unterrichts gezeigt werden. Neben der bequemen Übermittlung von Daten und der automatisierten Auswertung vorgefertigter Abfragen soll auch die Möglichkeiten gezielter Kurzbefragungen innerhalb von Unterrichtssequenzen diskutiert werden.

[Zoepfl.zip](#)