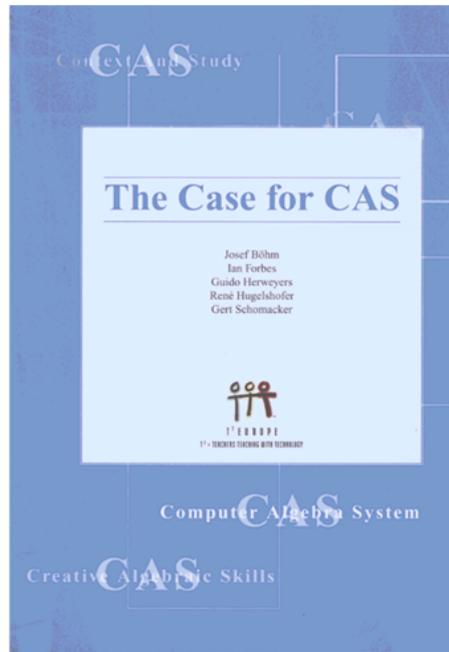


Teaching Aids for CAS-compliant Lessons

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Final exam at the vocational High school in Buchs (CH).
Students had no analysis.

Problem 1

- 1.1 Find the equation of a parabola going through the point $P(5/18)$ and which is touching the straight lines $g_1: y=x+4$ and $g_2: y=4x-2$
- 1.2 a second part of the problem, that I will omit here.

Student's average achievement was 75% of the possible points in this exercise. It was the best solved exercise.

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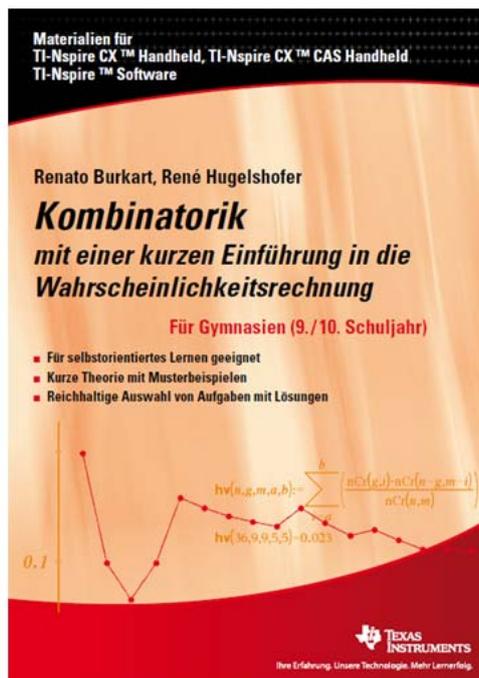
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All teaching materials are available as a PDF at (copying permitted, colour version):

www.ti-unterrichtsmaterialien.net (see introduction) with Problems as tns-files with solutions. Or printed (b/w) at www.ti-activities-shop.net

Combinatorics with Probability



Quadratic Functions & Equations



Emphasis on dynamic aspects

Ch 1: Parabolas incl. transformations (other functions as well). Focus. Real world exercises → in all the Scripts

Ch. 2: Solving equations

Tangent problems (very important).

Ch. 3: Extension of transformations on impicite functions.

Ch. 4: Extrem value problems

Some examples with the tangent method

Linear Functions

Ch 1: Linear functions, incl. piece-wise functions, normal, parameters.

Ch 2: System of equations, incl. Gauss algorithm, linear optimization



Linear Functions

General form $y = a \cdot x + b$

Often the translation form is better:

$$\frac{y - y_p}{x - x_p} = a \stackrel{\text{oder}}{\Leftrightarrow} y - y_p = a \cdot (x - x_p) \stackrel{\text{or}}{\Leftrightarrow} y = a \cdot (x - x_p) + y_p$$

And you have a first access to the rate of change.

Compare with

$$y - y_p = a \cdot (x - x_p)^2 \Leftrightarrow y = a \cdot (x - x_p)^2 + y_p$$
$$(x - x_p)^2 + (y - y_p)^2 = r^2$$

Functions and Models

Ch. 1: Potencies, formats (technical & scientific),
potency functions, transformations
-> example

Ch. 2: Exponential und logarithmic functions (many real world
problems)

Ch. 3: Sequences and series (incl. Cobweb und discrete growth
models)

Ch. 4: Modelling of dynamic systems

-> examples

Teaching Aids for CAS-compliant Lessons

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In this Workshop I will present you a series of booklets for students, which use CAS in a reasonable way. They cover the whole range of Algebra and Analysis for high schools. The main focus will be on the new book that you got in your folder. The Authors are Benno Frei, Robert Märki and myself. All the booklets can be downloaded for free as a PDF on the material server www.ti-unterrichtsmaterialien.net

Final exam question

Let me start with a task that was placed on the final exam of a vocational high school in Switzerland this year. It should be noted that vocational high schools in Switzerland usually have no Analysis.

Problem 1

1.1 Find the equation of a parabola that goes through the point P(5,18) and is touching the straight lines

$$g_1 : y = x + 4 \text{ and } g_2 : y = 4x - 2.$$

1.2 The parabolas $p_1 : y = \frac{1}{2}x^2 - 5x + 15$ and $p_2 : y = -\frac{1}{4}x^2 + 4x - 12$ touch each other at point B.

a) Calculate the coordinates of point B.

b) Determine the equation of the common tangent of the two parabolas in point B.

Student's average achievement for this problem was 75% of the possible points. It was the best solved problem.

The Students used our book “**Quadratische Gleichungen und Funktionen**” and they had a TI-Nspire CAS..

In this booklet you can find a method which I call “tangent method” and with which you can easily solve the above problem using CAS (see tns-file 1). With this method you can find axis of conics in an elementary way, e.g. the axes of a general parabola by just using quadratic equations (see tns-file 1).

If you want to have more information about this topic, I can send you a paper treating with it.

In any of the 4 booklets we have new aspects that show how you can treat Mathematics in a more efficient way.

The second book we produced was “Lineare Funktionen und Gleichungen” (Linear Functions and Equations). As in all the books the emphasis is on dynamic aspects. Therefore we start with functions and transformations of functions, rather than with equation solving. Usually teachers use the general form $y = ax + b$ for solving most problems. We prefer the slope form

$$\frac{y - y_1}{x - x_1} = a \Leftrightarrow y - y_1 = a \cdot (x - x_1) \Leftrightarrow y = a \cdot (x - x_1) + y_1$$

With that form you can solve most problems with only one equation. E.g. if you have a given Point and the slope of a straight line you get at once its equation. Or if you have two points you have directly the slope.

The first formula represents the important notion of “rate of change”. The second formula is the translation form of $y = a \cdot x$ through point (x_1, y_1) .

You will find in this book a lot of real world problems from different areas of application.

Before I start with the new book let me mention two other booklets:

Robert Märki: Differenzial und Integralrechnung (Analysis). An innovative introduction to Analysis based on the rate of change. Robert had presented this book last year at this Symposium.

R. Burkart, R. Hugelshofer: Kombinatorik (combinatorics with a first introduction to probability). We usually treat this subject in Switzerland in the 9th school year.

The new book

Our new book “**Funktionen und Modelle, kontinuierlich und diskret**” has 4 chapters: Potencies, exponential and logarithmic functions, sequences and series and mathematical modelling of dynamic systems.

Ch. 1: Potencies: Often teachers drill students in the first high school year with all kind of term manipulations. Very motivating for students? We prefer to learn the necessary skills when they are needed. Therefore we have a lot of exercises by hand for potencies in the 1st chapter. Why by hand? Because taping in formulas for potencies is time consuming and students often produce a big mess with the powers, because of the high position and the brackets.

But students can verify the results with CAS as the following example shows:

Problem 1.26 from the book (see tns-file 2)

Find all solutions of the equation $(x^2 + 5x + 5)^{x^2 - 25} = 1$.

Probably students will not find all the solutions, but by verifying with CAS (tns-file 2) they have to reconsider their solution and they learn much more by doing it themselves.

In Ch. 1 you will also find a lot of exercises with power of 10 (scientific or technical notation) and also transformations of power functions, including inverse functions.

Ch. 2: This is classical stuff, but with a lot of real world exercises.

Ch. 3 and 4: I want to concentrate on one aspect of these two chapters. How can you treat limited and logistic growth in a comprehensive way by introducing a limiting factor in linear and exponential functions?

Dumping linear and exponential functions (resp. arithmetic and geometric sequences)

Linear and exponential growth has a rate of change which is constant or proportional to the actual value.

In terms of sequences:

$$\frac{y - y_1}{x - x_1} = \frac{f(n) - f(n-1)}{n - (n-1)} = k$$

$$f(n) = f(n-1) + k$$

$$\frac{f(n) - f(n-1)}{n - (n-1)} = k \cdot f(n-1)$$

$$f(n) = f(n-1) + k \cdot f(n-1) = (1+k) \cdot f(n-1)$$

These sequences are not useful to describe growth processes as they tend to infinity. Therefore we

introduce a dumping factor with limit G : $\frac{G - f(n-1)}{G} = 1 - \frac{f(n-1)}{G}$

Limited Growth

$$f(n) = f(n-1) + k \cdot \left(1 - \frac{f(n-1)}{G}\right)$$

Take $G=1$:

$$f(n) = f(n-1) + k \cdot (1 - f(n-1))$$

The corresponding iteration function is

$$f(x) = x + k \cdot (1 - x)$$

Logistic growth

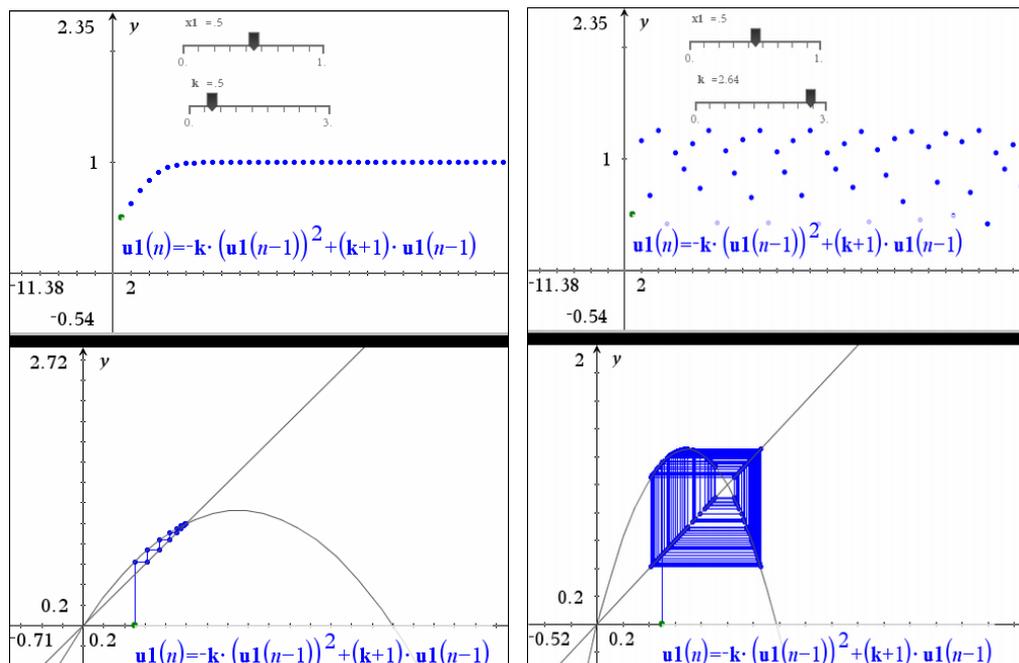
$$f(n) = (1+k) \cdot f(n-1) \cdot \left(1 - \frac{f(n-1)}{G}\right)$$

$$f(n) = (1+k) \cdot f(n-1) \cdot (1 - f(n-1))$$

$$f(x) = (1+k) \cdot x \cdot (1-x)$$

Problem 3.47 from the book (tns-file 4)

With the logistic growth you can get convergence to 1, periodic or chaotic behavior depending on the factor k . The reason for the different behavior can be seen nicely in the Cobweb presentation (fix point theorem).



We know this behaviour from the Feigenbaum diagram (factor k instead $k-1$).

The same can be done with the limited growth (problem 3.46).

In Ch. 4 you can find a modern treatment of dynamic modelling. First the modelling with adequate functions and then modelling of dynamic systems. You get a modelling system Dynasim for free. It's programmed in Java and therefore need no installation on your PC. And it is easy to handle (it will be improved in the near future and it is planed to translate it to LUA).

For time reason we make a simple example, that you can also do with a spreadsheet (not easy for students). Just to show you that dynamic systems programs are quite easy to handle. Such programs are used more and more in Science. The advantage of such programs is, that you can see visually at once when different dynamic problems have the same structure.

Problem (Example 4, p 103 in the book)

In a small pond there is a population of 500 fish (it could be any other population) which is increasing by 10% per year (birthrate). The capacity of the pond, confined by food, is 1000 fish. The deathrate is 3% of the actual population. Anglers want to use the pond for fishing. How does the fish population react on different catch quotas?

See file 4 (start first the program Dynasim)

