Technology and its integration in Mathematics Education

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July 2014. Krems, Austria

A stepwise CAS course for solving First-Order Partial Differential Equations

Gabriel Aguilera, José Luis Galán, M. Ángeles Galán, Yolanda Padilla, Pedro Rodríguez, Ricardo Rodríguez

Applied Mathematic

University of Málaga

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Introduction

❑ In most cases, the use of CAS is reduced to using computers as powerful high-performance calculators.

❑ It is therefore necessary to change the way people think about information technologies in order to optimise the opportunities they offer and try to encourage mathematical creativity among students.

Introduction

❑ Math teachers that use CAS have to change the traditional uses given to these tools.

❑ It is a mistake to use CAS in teaching as simple problem-solving machines.

❑ They should be used in ways that maximize the opportunities that these technologies offer: positively affecting student learning, significantly increasing opportunities for experimentation and allowing students to construct their own mathematical knowledge.

Introduction

❑ Math teachers must first set out a series of appropriate activities.

❑ The use of CAS in Mathematics has not reached optimum conditions. Mostly are blackbox (showing the result in one step without teaching students how to get there) and should be whitebox (showing intermediate steps).

Combining programming and CAS

❑ When students program, they must read, construct and refine strategies, modify previously written programs and lastly, use the programs to solve problems. This makes them the protagonists of their own learning.

❑ The most appropriate approach involves using programming and CAS together to allow students to create the specific necessary functions that will allow them to solve the problems involved in the subject matter under study.

Our experience

❑ Our experience over the last fifteen years has consisted mainly of continuously carrying out practical experiments in the computer laboratory in Mathematics courses for the degrees of *Technical Industrial Engineering* and *Technical Telecommunications Engineering* at the University of Málaga.

Our experience

Power of CAS $+$ Language program Flexibility $=$ Mathematical creativity

Programming with DERIVE:

elaboration of programs or specific functions by students

Software to use

❑ Among the great amount of mathematical software, we have chosen the program DERIVE for several reasons:

❑ Derive is easier to use than other mathematical programs as it operates with a very simple syntax.

❑ The student is capable of starting to solve problems by using the program in a short period of time, since basic functions and operations are available in several menus

❑ It needs few requirements, with regard either to memory and physical space, when it comes to installing it

Innovative aspect of our experience

❑ The main innovative aspect of these experiences is that the students have an active role in the sense they should elaborate themselves utility files to solve the typical problems for the different subjects. This fact implies that the students need to deal with programming in DERIVE, to understand the subject and to know how to solve typical problems.

Innovative aspect of our experience

❑ The fact that the students themselves are making the programs and including all of their arguments has a positive influence on their ability to subsequently apply them to solve specific exercises.

❑ Therefore, with this kind of activities CAS encourages active learning, greater comprehension of subject matter and mathematical creativity among students.

DERIVE 6 as a PeCAS

❑ The programs have been developed using the *Display* function in order to be used as didactical tools with explications of what the programs do step by step, using DERIVE 6 as a Pedagogical CAS (PeCAS) or as a white-box CAS.

First Order Partial Differential Equations

❑ Pfaff Equations:

   ❑ $P(x,y,z)\,dx + Q(x,y,z)\,dy + R(x,y,z)\,dz = 0$

❑ Quasi-Linear Equations:

   ❑ $P(x,y,z)\,p + Q(x,y,z)\,q = R(x,y,z)$

   ❑ Particular solutions

❑ Lagrange-Charpit Method

   ❑ $F(x,y,z,p,q) = 0$

Pfaff Equations
$P(x,y,z)\ dx + Q(x,y,z)\ dy + R(x,y,z)\ dz = 0$

❑ Particular cases:

   ❑ $P(x)\ dx + Q(x)\ dy + R(x)\ dz = 0$

   ❑ $P(x,y)\ dx + Q(x,y)\ dy + R(z)\ dz = 0$

   ❑ Exact equations.

❑ General Method:

1. Make 1 variable constant (z, for example) and solve $P(x,y,z)\ dx + Q(x,y,z)\ dy = 0$ (obtaining $G(x,y,z) = C$).

2. Find an integration factor $\mu(x,y,z)$.

3. Solve $dG + K(G,z)\ dz = 0$.

4. Substitute previous G to get the final solution.

Quasi-Linear Equations
P(x,y,z) p + Q(x,y,z) q = R(x,y,z)

❑ General solution:

    ❑ Requires finding integral combinations.

    ❑ Requires solving two first order ODEs or Pfaff Equations.

❑ Particular solutions:

    ❑ Requires finding the general solution.

    ❑ Requires solving a non-linear system of equations.

Lagrange-Charpit Method
F(x,y,z,p,q) = 0

❑   Particular cases:

    ❑ $F(p,q) = 0$

    ❑ $f(x,p) = g(y,q)$

    ❑ $z = px + qy + g(p,q)$

❑   General Method:

1.   Solve a first order ODE from a given system.

2.   Solve $p(x,y,z)$ and $q(x,y,z)$ from a non-linear system of equations

3.   Solve the Pfaff equation:

    $dz = p(x,y,z)\, dx + q(x,y,z)\, dy$

Contents of FOPDE.mth

❑ First order differential equations:

    ❑ Different types of differential equations (separable, homogeneous, exact, linear, Bernouille, Riccati, …). Generic programs. Cauchy's problems

❑ First order partial differential equations:

    ❑ Pfaff Equations.

    ❑ Quasi-Linear Equations and their particular solutions

    ❑ Lagrange-Charpit method for Partial Differential Equations of first order.
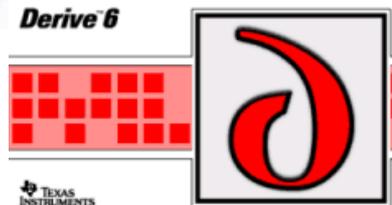
Example of a computer lecture on First-Order Partial Differential Equations

❑ Computer Lecture: Three blocks:

   ❑ Derive functions and developed programs

   ❑ Solved exercises

   ❑ Proposed exercises

❑ Solution with Derive

Other materials: Bologna Syllabus

- ❑ Preliminary concepts
- ❑ Theoretical aspects
- ❑ Exercises
- ❑ Questionnaire and its solution
- ❑ Forum
- ❑ <u>Videos</u>

Final Conclusions

❑ Our accumulated experience reveals that CAS are computer tools which are easy to use and useful in Mathematics courses for Engineering.

❑ The traditional uses given to CAS in the teaching of Mathematics for Engineering must be changed to maximize the opportunities offered by CAS technologies. Optimal use should be aimed at improving student motivation, autonomy and achieving participatory and student-centred learning.

Final Conclusions

❑ One powerful idea involves combining CAS resources with the flexibility of a programming language.

❑ There exists reasonable evidence to show that making programs with DERIVE facilitates learning and improves student motivation.

❑ Although it would be desirable to do so, it is not necessary to substantially modify the traditional program of studies of Math courses for Engineering to introduce the innovation of having students make their own programs with DERIVE.

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