Interactive Applications Utilization in Teaching Mathematics

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Abstract

Modern technologies have influenced the teaching of mathematics at primary schools, secondary schools, and universities. Nowadays, it is natural to utilise their advantages to reach teaching aims. It is widely known that mathematics should be taught to support students' active acquisition of knowledge. An appropriate use of interactive applications in teaching mathematics can contribute to this. In the paper, we present our own interactive applications that can be used in teaching mathematics at primary and secondary schools and universities. Moreover, we provide recommendations for their appropriate integration into education. Finally, we analyse our experience with integrating interactive applications into teaching mathematics at our faculty.

Keywords: Teaching mathematics, ICT in education, Interactive Applications

1 Introduction

Since the 1980s, modern technologies have influenced the teaching of mathematics at primary and secondary schools and universities. Initially, teachers tried to use computer programs to achieve a concrete educational goal. However, the importance of integrating modern technologies into education has gradually increased. Nowadays, it is natural that schools and universities use learning management systems to achieve educational goals.

There are many possibilities for how to integrate modern technologies into teaching. One of them is e-learning. There are many definitions of e-learning in the existing literature. By Wang et al. (2010), e-learning refers to the use of computer network technology, primarily over or through the Internet, to deliver information and instructions to individuals. However, most teachers who utilize modern technologies in education prefer a suitable combination of traditional face-to-face teaching and e-learning, making it possible to use the strengths of both teaching methods. In the scientific literature, this combination is referred to as blended

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learning. It is often considered by many authors to be an ideal way how to integrate modern technologies into education. There are many reasons why teachers prefer blended learning over other educational methods. For example, Graham and Dziuban (2008) identified six reasons for using blended learning: the richness of pedagogical stimuli, access to knowledge, presence of personal contact, own control over learning, cost-effectiveness, and ease of control. According to Graham (2006), adopting a blended learning approach improves learning effectiveness, access and flexibility, resulting in greater cost-effectiveness. Fisher (2003) considers this modern method of learning to be a selection of an optimum mix of instructional delivery strategies. According to Thorne (2003), blended learning is the learning agenda's most logical and natural evolution.

As for mathematics teaching, many studies prove the suitability of blended learning. The study of Malatinská et al. (2015) demonstrated that blended learning positively impacts both learning outcomes and on attitudes of students towards mathematics. Similar results were published by Lin, Tseng, and Chiang (2016). Moreno-Guerrero et al.'s (2020) research shows a positive influence on results, grades, motivation, autonomy, and participation. Another proof of blended learning suitability in mathematics teaching can be found in the works of Mišút and Pokorný (2015), Pokorný (2019), and Voštinár (2017).

The importance of integrating modern technologies into education was fully demonstrated during the recent COVID-19 pandemic. Schools were closed in many countries, including the Slovak Republic, and distance education remained the only possibility for educating pupils and students. However, neither teachers nor students were sufficiently prepared for such a sudden transition to distance education. Modern technologies proved to be the most suitable option for distance learning. Teachers began to widely use synchronous and asynchronous forms of online education, thanks to which they could eliminate the negative impact of the COVID-19 pandemic on the achievement of educational goals.

After the end of the COVID-19 pandemic, many teachers returned to face-to-face teaching and stopped integrating modern technologies into education. On the other hand, thanks to the pandemic, many teachers have learned how to use modern technologies appropriately to achieve educational goals and continue to use them even after the end of the pandemic. It can be concluded that the pandemic contributed to integrating modern technologies into education and taught teachers and students to benefit from their advantages.

2 Interactive Applications

2.1 Active Acquisition of Knowledge in Teaching Mathematics

A traditional teaching of mathematics, in which a teacher gives pupils and students facts and knowledge in the form of an explanation and then practices it with them, is often criticised. The pupils often acquire facts and knowledge only at the level of memorisation, without deeper understanding, and therefore cannot use it when necessary. Nowadays, mathematics

teachers should prefer methods that lead to the active acquisition of knowledge by pupils. It is indisputable that the activation of students is very important in mathematics teaching. We must remember this when integrating modern technologies into teaching mathematics. Žilková (2013) states that it is necessary to concentrate on active acquisition of knowledge. Hanzel (2013) stresses that dynamics and interactivity are required in teaching mathematics. Moreover, he states that electronic study materials from mathematics should not be written in a classical way (definition, theorem, proof, and example). According to Mišútová and Mišút (2012), the didactic effectiveness of teaching increases if students have at their disposal interactive online courses in addition to face-to-face teaching.

However, how can modern technologies be integrated into teaching mathematics to support the active acquisition of knowledge by pupils or students? One possibility is the use of interactive applications that will allow students to actively solve tasks and react to their mistakes in such a way which allows them to correct their incorrect solutions. However, there is a lack of didactically processed interactive applications that mathematics teachers and their students could use.

A degree of interactivity of educational materials is one of the problems. Tasks considered interactive by their authors often only check whether the student's result is correct or incorrect. However, when solving mathematical problems, not only the result is important, but also the procedure by which the student obtained the result. An interactive application does not mean that it only checks whether the result is correct or incorrect. The interactive application should help the student to find the correct result after his/her wrong solution.

In Slovakia, we can find many interactive applications on the Internet. Many of them are created by the teachers themselves. However, most of these applications only check the correctness of the student's answer. In addition, the numbers in the tasks are not randomly generated, so when you run them again, the result is always the same. In case of applications from foreign websites, there is a language barrier, and especially pupils and younger students cannot work with them. These factors then reduce the effectiveness of using interactive applications in teaching mathematics.

2.2 Interactive Applications for Primary School Pupils

This part of the paper introduces our interactive applications for primary school pupils. The first group consists of interactive applications for students aged 6 to 11 in the field of combinatorics. They are available at https://matematika.truni.sk/cvicenia?kombinatorika-1st-web.

In these applications, students look for all the possibilities of how to buy fruits, dress robots, prepare a snack, roll the desired sum on the dice, create pairs, colour balls, snowmen or flags, etc. An example of an interactive application is depicted in Figure 1.

In the application depicted in Figure 1, a pupil must find all six ways how to colour a flag so that it has one blue, one red and one white stripe. Pupils know that there are six possible ways, but they have to find them. Figure 1 also shows feedback after a pupil's incorrect

solution. Flags that are the same are marked with a blue cross, and flags that do not meet the conditions of the task are marked with a red cross.

Figure 1: Interactive application Colour flags.

Controlling the applications is simple and intuitive. Pupils get used to it in a few minutes. For example, we control the application in Figure 1 by clicking directly on the stripes on the flag, which cyclically change colours in the order white - blue - red - white... If a pupil wants to check the correctness of his/her solution, he/she clicks the Check button and gets immediate feedback about the correctness of his/her solution. In case of an incorrect solution, after viewing the feedback, the pupil can continue solving the task and correct his/her mistake.

The second series consists of 34 interactive applications for solving word problems. In them, pupils practice solving word problems for various tasks based on addition, subtraction, multiplication, and division.

The series is available at https://matematika.truni.sk/cvicenia?slovne-ulohy-1st-web. An example of the application can be seen in Figure 2.

The application in Figure 2 is aimed at solving multiplication word problems up to $5 \cdot 5$. In the task in the figure, pupils solve the word problem: "5 children came to lunch. Each of them got four buns. How many buns did they get together?"

At the beginning, pupils see only the task. After entering the correct result, pupils continue solving the next task. After entering an incorrect result, a picture with buns (see Figure 2) is displayed as feedback, thanks to which pupils can correct their solution. All numbers in the tasks are randomly generated. Moreover, a teacher can see how many tasks were solved correctly and how many mistakes were made.

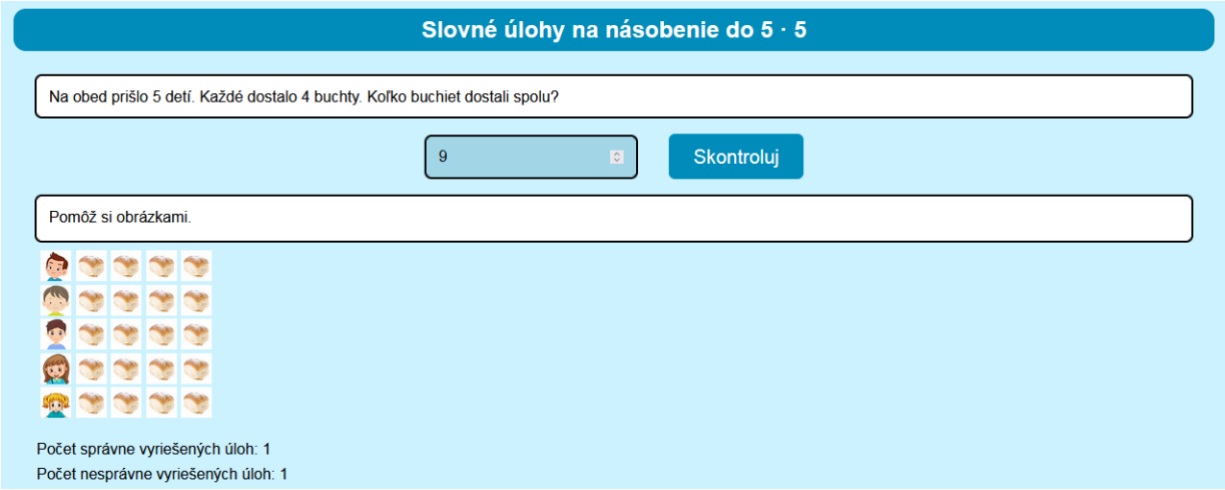

Figure 2: Interactive application for multiplication up to 5 5.

2.3 Interactive Applications for Secondary School Students

In this part of the paper, we introduce our interactive applications for secondary school pupils. The first group consists of 38 interactive applications aimed at teaching decimal numbers. They are available at https://matematika.truni.sk/cvicenia?desatinne-cisla-2st-web . The applications cover the content and performance standard determined in the Slovak State educational program. Thanks to these applications, students will learn to write and compare decimal numbers, round decimal numbers, add, subtract, multiply and divide decimal numbers, determine the value of numerical expressions, solve word problems with decimal numbers, convert units of length, area, volume and weight, represent numbers on the number line and determine the arithmetic mean of decimal numbers.

In Figure 3, we can see an example of the application in which a student has to determine a number marked on the number line. In case of an incorrect answer, students receive a hint shown in Figure 4, thanks to which they can correct their solution.

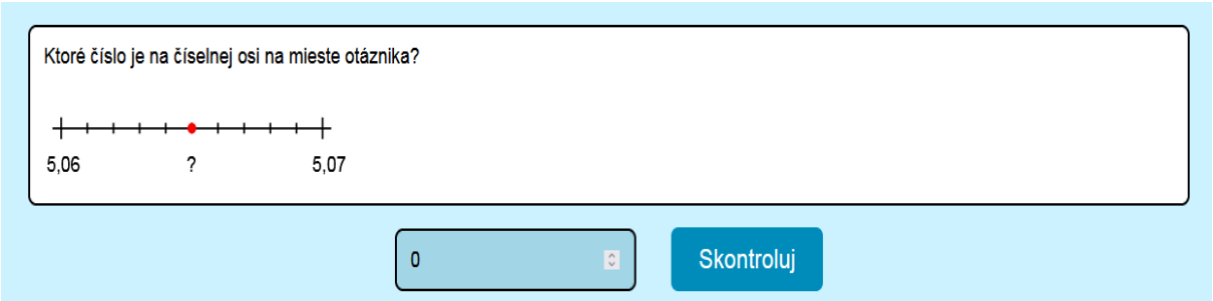

Figure 3: Interactive application Representation of decimal numbers on the number line.

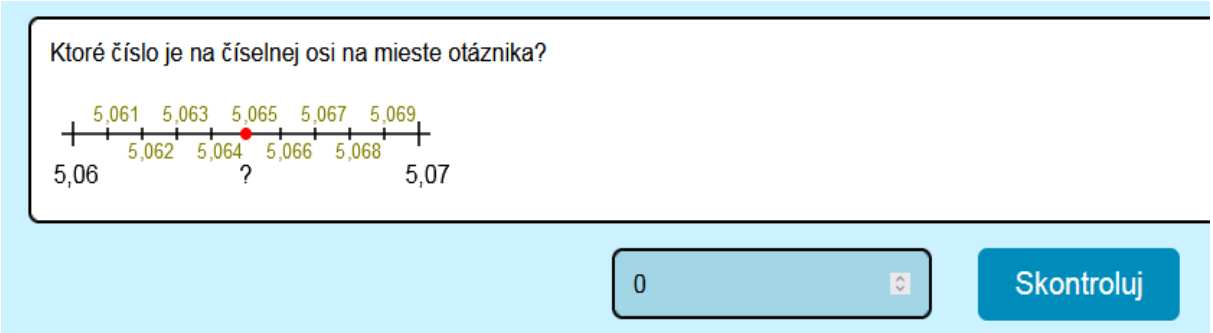

Figure 4: Feedback in the interactive application after an incorrect answer.

The second group consists of 19 interactive applications on the topic Variable, expression. They are available at https://matematika.truni.sk/cvicenia?premenna-vyraz-2st-web . Thanks to the applications, students learn to determine the value of an expression with one and two variables, use a formula to calculate the value of a variable, express a relationship between variables, express an unknown from a formula, add and subtract expressions with one and two variables, multiply an expression in parentheses by a positive and negative number, determine the coordinates of the point in the coordinate system and mark the point in the coordinate system.

The third group consists of 24 interactive applications on the topic Ratio, Direct proportionality, and Inverse proportionality. They are available at https://matematika.truni.sk/cvicenia?pomer-2st-web . Thanks to the applications, students learn to write a given ratio in equivalent forms, increase or decrease a quantity in a given ratio, divide a quantity in a given ratio, and solve word problems with ratio, direct proportionality, and inverse proportionality.

2.4 Integration of Interactive Applications into Teaching

In the previous part, we describe five groups of interactive applications for teaching mathematics of pupils aged 6 to 15. In our opinion, the advantages of integrating the applications into teaching mathematics are:

1. Applications have a wide range of utilization. Firstly, it is possible to use them directly during a lesson. For example, on an interactive whiteboard, a teacher can use an application depicted in Figure 1 to demonstrate a suitable way of solving the given combinatorial task (for example, both flags in the first row have the first stripe blue, both flags in the second row have the first stripe red, both flags in the third row have the first stripe white). Secondly, pupils can work with applications on tablets or notebooks during the lesson. Thirdly, pupils can work with applications in the school children's club, during a lesson with substitute teacher, during preparation at home, etc. Of course, we recommend that when working with applications, each pupil has his/her own device.

2. As we have already mentioned, it is known that pupils learn much more if they acquire knowledge by an active way, not by writing down solutions of classmates or the teacher. If pupils are working on their own notebook or tablet, they are forced to work and solve problems actively.
3. The applications provide pupils with immediate feedback about the correctness of their solution during their work. If pupils' solution is incorrect, they can correct it. We consider this to be an advantage, especially when a pupil is doing homework since the application itself checks the correctness of the homework solution (there is no such feedback when solving the homework in a printed worksheet).
4. The applications support the pupil's own pace of work. Each pupil works at a different speed; some only solve the third task, and others are already on the seventh. In this way, it is achieved that the slower students catch up, while the faster ones do not have to wait for anything and are also active all the time.
5. Applications support an individual approach to pupils. For example, a pupil with trouble with addition up to 10 can work with this application, while another pupil may already be working with an application focused on addition up to 20.
6. Applications can be run on many different devices (computers, laptops, tablets, and mobile phones) with Windows, macOS, Android, iOS and other operating systems with supported browsers.
7. As mentioned earlier, the applications randomly generate numbers in the word problems, so the results do not repeat when returning them. So, it is not enough to remember that the task with the buns from Figure 2 has a consequence of 20 because when you start it again, the numbers (and thus also the result) could be different.
8. The applications can be used during a group work of pupils. For example, a pupil who no longer has a problem with the application works with a classmate and helps him/her.

2.5 Interactive Applications for University Students

At our faculty, we prepare future teachers at primary and secondary schools. The largest group of student’s study Pre-School Elementary Pedagogy. After their bachelor's, they can become teachers at kindergarten or continue a master's study in Primary Education Teaching to become teachers at primary schools.

They must complete the course Combinatorics and Working with Data during their study. The course is taught by blended learning, a combination of face-to-face lessons with a teacher and an e-learning course in Moodle. There are two groups of students. Full-time students have one face-to-face lesson every week; part-time students have an introductory online lesson and an online consultation at the end of the term. In a group of full-time students, we use a flipped classroom. In Moodle, students have video lessons, and every week, they have to study one of them before a face-to-face seminar, where the content of the video lesson is practised. A video recording of a workshop is available in Moodle for both full-time and part-time students for further use.

To make our students more active, we prepared a series of 62 interactive applications from combinatorics (one of them is depicted in Figure 5), four applications from probability and seven applications from descriptive statistics. The applications are available for students in Moodle. The combinatorics applications focus on the systematic solutions of combinatorial tasks by finding all possible combinations, variations, and permutations. Thanks to the applications, students have immediate feedback about their knowledge level after studying video lessons in Moodle.

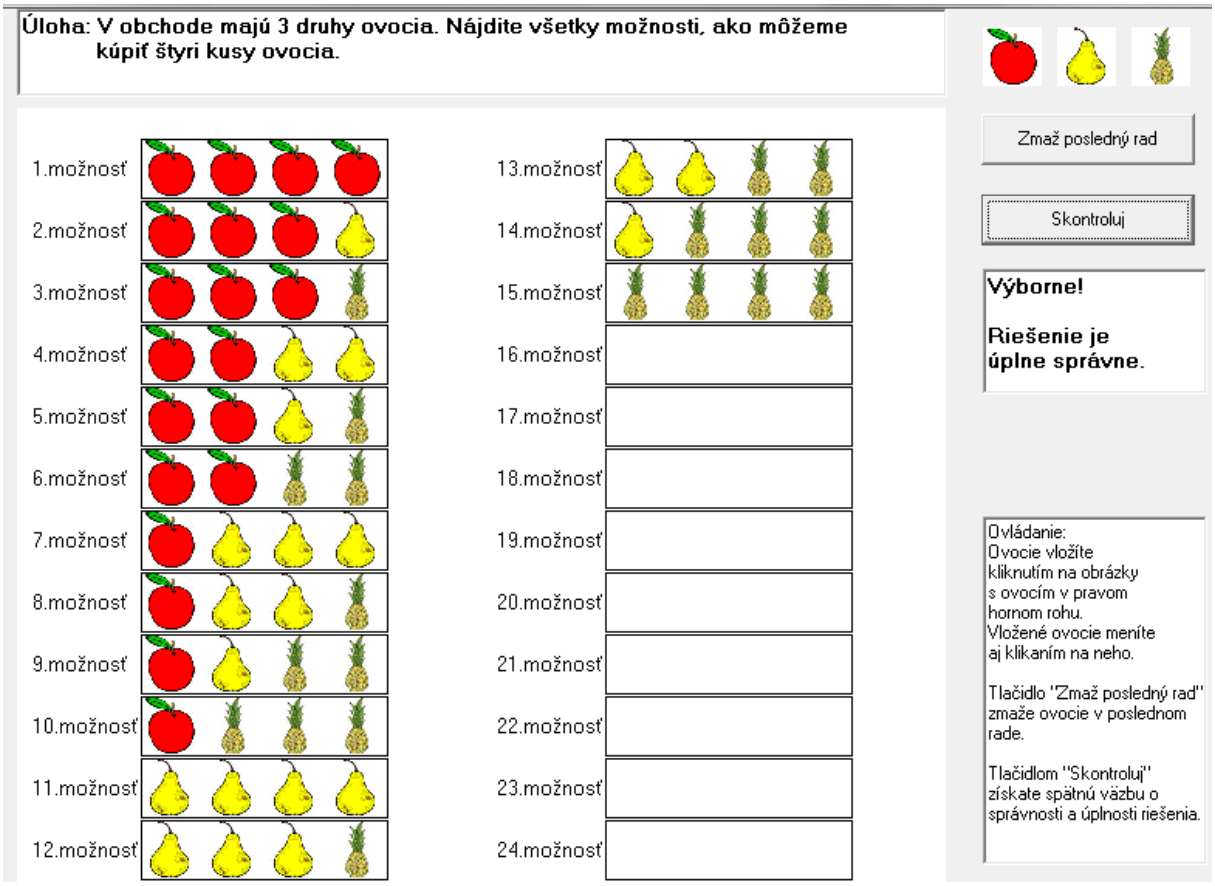

Figure 5: Example of an interactive application from combinatorics.

In an interactive application in Figure 5, the students have to solve the following problem. The store sells apples, pears and pineapples. Find all the possibilities of how four pieces of fruit can be bought. As we can see, there are many possibilities for finding the correct solution. The application recognises all correct solutions. If the solution is incorrect (some possibilities are missing, some of them are duplicates), students have immediate feedback, and they can correct their solution as many times as necessary.

To determine students' opinions on the teaching method mentioned above, we used a questionnaire, which 97 full-time and 167 part-time students filled out.

In the first question, we investigate whether our students consider the video lessons in Moodle to be helpful. Students chose an answer from 1 to 5, where one meant "not useful at

all" and five meant "very useful". Out of 97 full-time students, 90 chose option 5, 5 chose option 4, and only one chose options 1 and 2. Out of 167 part-time students, 160 chose option 5, 6 chose option 4, and only one chose option 1.

In the second question, we investigate whether our students consider the course in Moodle with interactive applications to be useful. Again, the students chose an answer from 1 (not useful at all) to 5 (very useful). Out of 97 full-time students, 76 chose option 5, 13 chose option 4, 6 chose option 3, and only two students chose option 2. Out of 167 part-time students, 148 chose option 5, 15 chose option 4, and only three students chose option 3.

In the third question, we investigate whether our students consider the text documents (in pdf or docx format) in our course in Moodle to be useful. Out of 97 full-time students, 73 chose option 5, 14 chose option 4, 8 chose option 3, and only one student chose option 2. Out of 167 part-time students, 141 chose option 5, 12 chose option 4, 11 chose option 3, and only two students chose option 2.

In the fourth question, we investigate whether our students consider their final evaluation from Combinatorics and Data Processing to be objective. Students chose an answer from 1 (it is not objective at all) to 5 (it is really objective). Out of 97 full-time students, 80 chose option 5, 11 chose option 4, and 6 chose option 3. Out of 167 part-time students, 149 chose option 5, 11 chose option 4, 4 chose option 3, and only one chose options 1 and 2.

In the fifth question, we investigate whether our students are satisfied with their final evaluation from Combinatorics and Data Processing. Students chose an answer from 1 (I am not satisfied at all) to 5 (I am very satisfied). Out of 97 full-time students, 80 chose option 5, 6 chose option 4, 10 chose option 3, and only one student chose option 2. Out of 167 part-time students, 133 chose option 5, 18 chose option 4, 9 chose option 3, 2 chose option 2, and three chose option 1.

Finally, in the sixth question, the students were asked to rate a teaching of Combinatorics and Data Processing. They had to use a scale from A (best grade) to FX. Out of 97 full-time students, 65 chose option A, 29 chose option B, 2 chose option C, and only 1 student chose option D. Out of 167 part-time students, 147 chose option A, 13 chose option B, 6 chose option C, and only one student chose option D. No student chose options E or FX.

3 Conclusion

Nowadays, our children use mobile phones and tablets daily. Computer rooms, interactive whiteboards and tablets are gradually increasing in our schools. Therefore, it is natural to use these devices in teaching mathematics, starting from the first grade of primary school. However, it is necessary to mention that these devices do not automatically improve the educational process. There is a need for high-quality didactic software which will contribute to the achievement of educational goals.

There are many interactive applications for teaching mathematics on the Internet. However, most of them are not in the Slovak language, which causes a language barrier for

our primary and secondary school students. Naturally, it is impossible to assume that they fully correspond to the requirements in the Slovak State educational programme. We are convinced that our applications described above could contribute to the efficient achievement of teaching aims since they make pupils and students achieve knowledge in an active way. Moreover, working with our applications respects the student's own pace and level of knowledge. Unfortunately, we did not have an opportunity to test the efficiency of the above-mentioned applications at primary and secondary schools. However, we tested the effectiveness of our older applications, which positively impacted not only pupils' knowledge but also their positive attitude towards mathematics (see Malatinská et al., 2015). Therefore, we welcome collaborating with primary and secondary school teachers on implementing such research.

We know that our applications are currently only available in Slovak. We plan to add English or Czech versions in the future.

As for the utilisation of interactive applications in teaching mathematics at our faculty, their integration into education has brought several positives. As we have mentioned above, the majority of both full-time and part-time students were really satisfied with our method of teaching, which combines video lessons, texts, interactive applications and face-to-face teaching. Nearly all students agreed that the Moodle course with interactive applications was helpful. Although video lessons were the most popular, interactive applications were also highly valued. Our research also revealed a positive impact on students' feelings of objectiveness in their final evaluation. We state that this is also due to the use of interactive applications that give students accurate and reliable feedback about their level of knowledge. It is also necessary to mention that most students are satisfied with their final assessment, which is not always obvious when teaching mathematical subjects. Finally, the evaluation of our teaching method using interactive applications by our students, where more than 80 per cent of students used the best grade A, shows their satisfaction. Let us note that such assessment is not usual in teaching mathematical subjects.

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