Prospective Secondary Mathematics Teachers' Use of CAS-based Textbook Elements

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Abstract:

This study investigated how a group of 10 prospective secondary mathematics teachers (PST) read, evaluated, and adapted a textbook lesson involving the symbolic manipulation capabilities of computer algebra systems (CAS-S). PST read the entire lesson and tended to focus on the organizing question at the beginning of the student lesson and the CAS-S sections of the lesson. PST frequently evaluated the lesson with respect to the teacher and whether lesson elements would promote student understanding. Only one PST evaluated the lesson with respect to the student and how they student might interpret lesson elements. There were five categories of curricular adaptations exhibited by PST: following, additive, mix, reductive, and adaptive. In general, the adaptations made to the lesson by PST were positive. CAS-S based elements were occasionally removed by PST but typically due to non-technology based reasons such as problem redundancy. Several positive adaptations were made by PST to CAS-S based elements such as asking students to make predictions before using CAS-S and understanding the hidden procedures used by the technology.

Key Words: prospective secondary mathematics teachers; computer algebra systems; symbolic manipulation, textbooks

Introduction

Textbooks strongly influence instruction in secondary mathematics classrooms in the U.S. (Smith, 2013). For a number of years activities involving graphing calculator technology have appeared in U.S. secondary mathematics textbooks (e.g., Holiday, et al., 2005). Recently, however, three secondary mathematics textbook programs have appeared that incorporate computer algebra system (CAS) technology. It is certainly important to understand how practicing teachers use CAS-based lesson elements in their lesson planning and classroom lessons, however, prospective secondary mathematics teachers (PST) differ from practicing teachers. In order to inform the design and implementation of university methods courses that are intended to prepare PST for using CAS in their future mathematics classrooms it is important to understand how this group of novice teachers uses these elements. Consequently, this study examined how ten PST interacted with CAS-based elements in a U.S. reform-oriented secondary mathematics textbook lesson.

Background

Before embarking on a description of pertinent research with regard to teachers, CAS in curriculum, and prospective teachers' use of curriculum I will begin by defining terminology that will be used throughout this paper. A computer algebra system or CAS can reside on a variety of different platforms (e.g., handheld calculator) and consists of graphical, numerical, symbolic, and tabular representational systems that are connected and often dynamically linked together. I use the acronym CAS-S to refer specifically to the symbolic manipulation capabilities of a CAS. Curriculum will refer to written or electronic classroom activities that may or may not provide suggestions to the teacher for how they can be implemented.

Teachers' Use of CAS

Previous research conducted by Kendal and Stacey (2001) on two practicing teachers (Andre and Benoit) who were attempting to integrate CAS into their secondary mathematics classrooms suggests that a variety of beliefs influence teachers' use of CAS. Andre believed that mathematics primarily consisted of rules and used the CAS to find the exact gradient for the tangent to a curve. He also believed strongly in a lecture style of teaching and, as a result, provided flowcharts to students so that they could use the CAS. Benoit, in contrast believed that mathematics consisted primarily of conceptual understanding. CAS was used in his classroom in accordance with this belief by helping students to connect the numerical and graphical representations for mathematical ideas and less on algebraic manipulation of symbolic forms.

Özgün-Koca (2010) examined the beliefs of prospective teachers with regard to CAS. A total of 59% of teachers in her study did not believe that CAS-S would be beneficial in helping students to learn algebra before an intervention involving the CAS-S as black box, gray box (Cedillo & Kieran, 2003), and a symbolic math guide (SMG). After the intervention, 48% of the prospective teachers indicated their beliefs with regard to the CAS had changed. While prospective teachers didn't like the use of CAS as a black box they did indicate that it could be used once students had mastered paper-and-pencil procedures. The teachers also saw advantages to the use of the CAS as a gray box and the use of the SMG as a means by which their future students could learn symbolic manipulation.

Incorporating CAS-S in Curriculum Resources

Heid and Edwards (2001) describe four different roles in which CAS-S can be incorporated within curricula. First, the technology can be used as the primary producer of symbolic results allowing students to focus on conceptual understanding of mathematical ideas or problem-solving activities. Second, the technology can provide students with opportunities to solve equations step-by-step in what Cedillo and Kieran (2003) describe as a gray box approach. Third, CAS-S can be used to generate examples from which patterns can be detected and conjectures formulated as seen in Figure 1. Fourth, Heid and Edwards (2001) describe how CAS-S can be used to locate formulas such as the quadratic formula as seen in Figure 2. CAS-S can also be used to complete one or more steps of a mathematical proof (Garry, 2003).

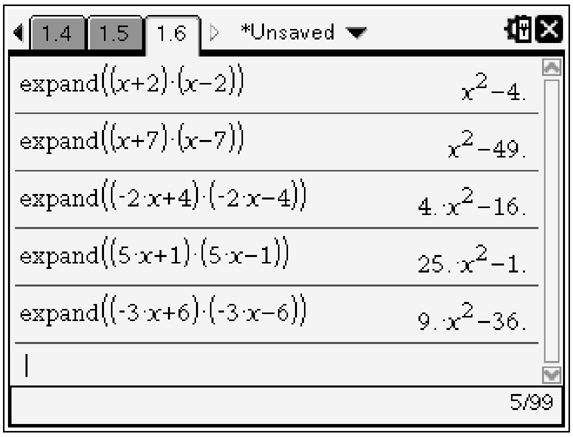

Figure 1. Using CAS-S to generate examples from which patterns and conjectures can be formulated.

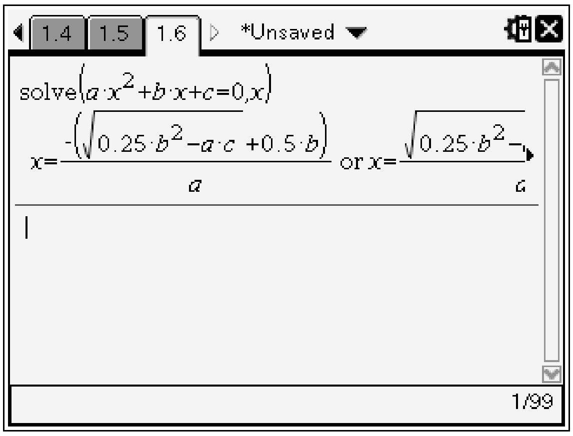

Figure 2. Using CAS-S to locate or develop general formulas.

Teachers' Interactions with Textbooks

A number of different factors have been identified that influence how both prospective and practicing teachers interact with textbooks (Stein, Remillard, & Smith, 2007). These factors include the following: beliefs and knowledge about mathematics and how it should be learned; orientation towards curriculum; professional identity, perception of students and their abilities; time; local context; teacher support; and specific curriculum features.

Remillard and Bryans (2004) observed the mathematics textbook interactions of eight primary teachers. They examined teachers' use of their textbook and placed them into three categories: intermittent and narrow; adopting and adapting; and thorough piloting. Teachers whose use was intermittent and narrow only exhibited a minimal use of the curriculum and relied on their own strategies or resources to design and teach classroom lessons. Adopting and adapting teachers used the materials as a guide for the general structure and content of their lessons, but used their own strategies to enact lesson content. Teachers who thoroughly piloted the curriculum used the textbook as their primary guide and used the suggestions in the teacher resource materials to implement the textbook lessons.

They also developed the construct of orientation toward the curriculum as the collection of perspectives that teachers' possess about mathematics, teaching, learning, and curriculum that influence how they interact with their textbook resources. Several new teachers followed the curriculum closely and were described as possessing a piloting orientation. Other teachers' curriculum orientations ranged from adherent and trusting to skeptical.

Sherin and Drake (2009) developed a curriculum strategy matrix to examine how primary school teachers interacted with their textbooks. Each row of the matrix represents a time frame with regard to the classroom lesson: before instruction, during instruction, and after instruction. Each column of the matrix represents a different activity that teachers can engage in with respect to textbook materials. In a group of ten elementary school teachers they found that teachers either read for a general overview of the textbook lesson or for detail before instruction and for detail during the lesson. No teachers read the textbook lesson after instruction. Teachers evaluated the textbook lesson for themselves as teachers, students, or for the others such as parents. Teachers adapted the textbook materials by omitting, replacing, or creating textbook activities.

In sum, there are a number of different ways that CAS-S can be used in textbooks. Additionally, research suggests that practicing teachers' beliefs about mathematics and how it should be taught influence their classroom use of the technology. Classroom interventions involving CAS-S have been found to be helpful in changing prospective teachers' beliefs about the classroom efficacy of this technology. U.S. secondary mathematics teachers use curriculum frequently and research indicates that a number of factors influence this use. A helpful framework for analyzing how teachers interact with textbook resources is presented by Sherin and Drake (2009) who consider how teachers read, evaluate, and adapt textbook lesson elements before, during, and after instruction. Despite this research, we still know little about how prospective teachers interact with CAS-S based textbook elements. This study seeks to answer the following research question: How does a group of PST read, evaluate, and adapt elements of a reform-oriented lesson infused with tabular, graphical, and CAS-S representational forms?

Methods

A total of ten PST enrolled in a technology methods course at a large university in the Midwestern portion of the United States participated in the study. Both secondary mathematics minors (requiring 28 credits in mathematics) and secondary mathematics majors (requiring 41 credits in mathematics) are required to successfully enroll in this class. Information regarding this group of PST is shown in Table 1. The first four weeks of class were devoted to secondary mathematics instruction using the Texas Instruments TI-83 and 84+ calculators. The last twelve weeks of class were used to provide PST with opportunities to learn mathematics and reflect on mathematics pedagogy with the Texas Instruments TI-Nspire CX CAS handheld. These TI-Nspire CX CAS activities were situated within the function content strand with a few activities involving number. During the last twelve weeks of class PST were asked to evaluate elements within three different textbook lessons that incorporated CAS-S to varying degrees. This paper will focus on the first of these three textbook lesson evaluations involving the first course in the *Core-Plus Mathematics* (CPM) program (Hirsch et al., 2008).

Table 1: *PST Participants*

PST (Pseudonym)	Gender	Math Major/Minor	Math Grade Point Average (out of 4)	Math Education Grade Point Average (out of 4)
Alan	Male	Major	3.29	3.50
Gary	Male	Minor	2.35	3.00
Jason	Male	Minor	3.03	2.75
David	Male	Major	2.91	3.50
Cathy	Female	Minor	3.28	4.00
Marcia	Female	Major	4.00	4.00
Steve	Male	Major	3.33	3.50
Chris	Male	Major	3.85	3.67
Bart	Male	Major	2.55	3.00
Ethan	Male	Major	4.00	3.50

CPM Textbook

The CPM program is a four-year program designed using the U.S. mathematics reform document, *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000), as a blueprint. This textbook uses real-world contexts to motivate and introduce students to a mathematical ideas embedded within four different mathematics content strands: algebra and function; geometry and trigonometry; statistics and probability; and discrete mathematics. The lesson analyzed was located in the algebra and functions strand of the first year of the program. Each CPM contains an investigation, a set of summarizing questions (Summarize the Mathematics [StM]), and a set of questions that provide students with opportunities to practice the main ideas of the lesson (Check Your Understanding [CyU]). A lesson element was defined as one problem that was denoted in the teacher's edition of the textbook with a separate solution. It was not unusual for one numbered task in the textbook to denote two lesson elements as it contained two questions. Enumerated in this way the CPM lesson contained a total of 28 lesson elements.

In the investigation students are introduced to a formula to predict the income I for a given price p at a bungee jump carnival ride: $I = p(50 - p)$. Students are asked to use table representations to solve equations for income and price and inequalities for price. Students use graphical representations to solve for price and income, describe how income changes as price increases, and to locate the price that will yield the maximum income. Students are introduced to the "|" and the solve commands in order to find the income given certain prices and to find a price for a given income, respectively. Students are also asked to think about how the equation $0 = p(50 - p)$ can be solved using paper-and-pencil (PP) techniques. In the StM and the CYU, students are given the rational and linear equations, respectively, and asked to solve for both the independent and dependent variables using tables, graphs, and CAS-S. It is interesting that only when students are working with a CAS-S are they asked to check their work; none of the work involving other representations requires checking. In addition, the last tool students are asked to use to solve equations is the CAS-S. Thus, the authors of the textbook appear to privilege the table and graph over the CAS-S.

Data Collection and Analysis

This study used two different frameworks. First, research such as Heid and Edwards (2000) led to a framework to categorize the use of CAS-S within the textbook lesson as well as how PST intended to use CAS-S based lesson elements as described in their adaptation logs. Categories of CAS-S within the CPM lesson were enumerated and other categorizes were developed as needed. In addition, hidden messages associated with CAS-S and other representation (e.g., table) use were also identified in the CPM lesson.

Second, Sherin and Drake's (2009) curriculum strategy framework as a way to frame the data collection activities and analyses associated with PST' interactions with the CPM lesson. However, instead of asking prospective teachers how they engage in reading, evaluating, and adapting before, during, and after instruction this study only gathered data on prospective teachers' reading, evaluating, and adapting before instruction.

Each prospective teacher was provided with two different highlighters, yellow and another color. PST were asked to use the yellow highlighter to note the parts of the textbook lesson that were important and the other highlighter to indicate what they had read. Prospective teachers were asked to write on the photocopied version of their textbook lessons to indicate what parts of the materials they had evaluated. Photocopies were collected and analyzed for nine PST participants. In order to understand what elements of the textbook lessons were adapted PST were asked to complete an adaptation log where each element of the lesson appeared in the first column. PST were asked to determine if they were to keep, adapt, supplement, or omit this textbook element and why as seen in Table 2. Adaptation logs were analyzed for all ten PST.

Table 2: *PST Adaptation Log*

Lesson Element	Curricular Adjustment	Justification
#3b	Supplement	This problem is useful as long as the students can somehow relate it back to the expression or check their answers by hand to show that they understand how the answers work within the expression and outside of the calculator and its method of calculation.

PST chose *keep* if they wished to retain the lesson element as originally described in the textbook lesson. PST used *adapt* to denote their intention to keep the intent of the lesson element, but change it in some way. The supplement curricular adjustment indicated that teachers wished to keep the lesson element but add some component to it. PST chose omit if they wished to remove the curricular element.

The frequencies of these four different components were enumerated for each of the ten PST participating in the study. Frequencies among these components were used to place PST into categories. PST justifications for each element were coded using in-vivo coding. Individual codes were grouped and those codes that occurred most frequently were reported. Themes were generated using qualitative data analysis methods of analytic induction and constant comparison (Miles et al., 2014).

Highlighted photocopies of the CPM textbook lesson were analyzed to determine what the PST read and thought were important. Highlighted lesson elements were categorized and enumerated. Sherin and Drake (2009) categorized practicing teachers' evaluations in terms of the target audience. PST evaluations were categorized first in terms of the intended audience and later categorized in terms of the nature of those evaluations.

The nature of the changes made to the curriculum (adapt, supplement, and omit) were categorized in terms of whether they were positive, neutral, or negative. Positive changes were those aligned with the CPM lesson goals, aligned with reform-oriented principles, or provided students with more opportunities to use technology. Negative lesson alterations were those that did not meet one or more of these criteria. Neutral elements were those that contained both positive and negative elements.

Results

Categorizing Uses of CAS-S in CPM Lesson

Out of a total of 28 lesson elements in the CPM lesson, eleven of these asked students to use the CAS-S tool. Out of this eleven three used CAS-S for symbolic manipulation and asked students to use a table, graph, or CAS-S to check their work. Two other lesson elements used CAS-S for symbolic manipulation and asked students to use tables, graphs, PP, or CAS-S to check the answers given to them by the technology. Five lesson elements asked students to use CAS-S for symbolic manipulation without asking them to check their answers. One lesson element asked students to reflect on the strengths and weaknesses of the CAS-S and other tools such as table, graph, and PP.

Read

All of the PST read the CPM lesson completely. However, there was variation in what the PST felt were important in the textbook lesson. Gary highlighted the highest number of questions as 13 while Cathy and Marcia did not highlight any questions. The majority (7/10) of PST felt that the advanced organizer worded in the form of a question at the beginning of the lesson was important. That is, the PST used this as a goal for the lesson and considered it important to their understanding of the lesson. All of the PST noted components of the lesson associated with the CAS-S tool. Moreover, seven out of ten PST noted that the fact that CAS-S can produce answers in both approximate and exact forms was important. Only five and four of the PST highlighted components of the lesson associated with the table and graph, respectively.

Table 3: *Important Components of the CPM Lesson*

			Technology		
PST	Advanced Organizer	Questions	Table	Graph	CAS-S
Alan	No	7/21	Yes	No	Yes
Gary	Yes	13/21	Yes	No	Yes
Jason	Yes	10/21	No	No	Yes
David	Yes	4/21	No	No	Yes
Cathy	Yes	0/21	Yes	Yes	Yes
Marcia	Yes	0/21	Yes	Yes	Yes
Steve	Yes	6/21	No	Yes	Yes
Chris	No	8/21	No	No	Yes
Bart	Yes	5/17[1]	Yes	Yes	Yes

Evaluate

The comments that PST wrote on their photocopied versions of the CPM lesson were used to understand how they evaluated the textbook materials. Most of the PST evaluated

[1] Four questions were missing from the photocopied textbook lesson.

the CPM lesson from the perspective of a teacher. Alan, Cathy, Marcia, and Chris completed problems as part of their evaluations. This work could be from either a student or a teacher perspective. Given this possibility, Chris was the only PST who evaluated materials solely from the perspective of the student. For instance, he read several lesson elements and wondered if students would understand these components after reading them. Although asked to do so, two PST did not evaluate the lesson materials at all. The most frequent category was evaluating lesson elements from the teacher's perspective to determine if they promoted student understanding.

Adaptations

The frequencies of CPM lesson elements across PST and the categorization of PST on the basis of these adaptations are shown in Table 4. Three PST followed the curriculum quite closely and were assigned the category *following*. Cathy and Steve suggested adding a number of elements to the curriculum and were consequently categorized as *additive*. Bart added a number of lesson elements, but also omitted a number of elements. For this reason he was categorized as *mix*. Ethan and Jason eliminated a number of different elements and was categorized as *reductive*. Marcia made a number of alterations to the CPM textbook lesson and was categorized as *adaptive*.

Table 4: *PST Lesson Element Adaptations and Categorizations*

PST	Keep	Omit	Adapt	Supplement	Curriculum Use Designation
Gary	**25**	1	0	2	Following
David	**21**	0	3	4	Following
Alan	**19**	1	8	0	Following
Cathy	12	0	2	**14**	Additive
Steve	13	3	2	**10**	Additive
Chris	15	0	5	**8**	Additive
Bart	13	**7**	1	**7**	Mix
Ethan	16	**8**	4	0	Reductive
Jason	18	**5**	1	4	Reductive
Marcia	2	2	**22**	2	Adaptive

Recall that the explanations that PST provided were categorized. In general, participants did not have similar reasons for keeping, adapting, supplementing, and omitting lesson elements either within or across curriculum use categories (e.g., following). For example, Alan kept items primarily because they promoted different ways of solving problems, adapted lesson elements because of questionable clarity, and omitted items that he felt were redundant in some way. There were four exceptions as noted in Table 5. David and Steve interacted with the curriculum due to mathematics content-based reasons. Chris interacted with the CPM lesson around strengths and weaknesses of tools. Bart curriculum interactions were focused on students' understanding technology.

Table 5: *Explanation Categories by PST and Curriculum Use Designation*

Curriculum Use Designation	PST	Curriculum Category	Explanation Category
Following	David	Keep	Mathematics Content
		Adapt	Mathematics Content
		Supplement	Mathematics Content
Additive	Steve	Keep	Mathematics Content
		Adapt	Cognitive Demand Students Choose Tool
		Supplement	Mathematics Content
	Chris	Keep	Strengths and Weaknesses of Tools
		Adapt	Strengths and Weaknesses of Tools
		Supplement	--
Mix	Bart	Keep	Student Explanation Understand Technology
		Adapt	Students Choose Tool Student Explanation
		Supplement	Understand Technology
		Omit	Understand Technology

Nature of Adaptations

The breakdown of changes by PST are shown in Table 6. The majority of PST made positive changes to the CPM lesson. Give examples of how they changed the curriculum in positive ways. A common positive change to the CPM lesson was supplementing a question by asking that students explain their thinking. Another common positive change to the lesson was increasing the cognitive demand of questions. For instance, one of the questions in the homework section at the end was worded in the following way:

> To find the **break-even point** for the business, you need to find the value of n that produces a value of P equal to 0. That means you have to solve the equation $0 = 6.5n – 2{,}500$. What values of n satisfy that equation? (Hirsch et al., 2008, p. 56)

Several PST adapted this question by removing the equation $0 = 6.5n – 2{,}500$ as they felt that this was too leading and lowered the cognitive demand of the question. Other PST added questions that provided students with opportunities to learn new technological functions such as using the *calculate maximum* command instead of just tracing a graph to locate the maximum.

Neutral changes to the curriculum involved changes to the wording of the problem that did not change the intent of the problem. Also several PST allowed students to choose their own tool (graph, table, or CAS-S) to solve the homework problems. This change was considered positive in that students were given a choice of tools, but potentially negative as they would be given less practice with the other two tools that were not chosen.

Two PST made primarily negative changes to the curriculum, Jason and Ethan. Both Jason and Ethan eliminated a number of lesson elements. For instance, Jason asked students to use a graph to find the income for a given price, but then eliminated a question where students were asked to trace along a graph to find the price for a given income. He justified omitting this question because he felt it was redundant. While the technological knowledge applied is similar in both cases the mathematical knowledge that is being tapped here is different. Ethan also eliminated questions because he felt that they were redundant even though they required students to use different mathematics content knowledge. Ethan did not like that students were asked to solve problems in the summary section that were not connected to a context. He felt that they did not promote understanding for students if they were to solve them.

Table 6: *Nature of Adaptations by PST*

PST	Positive	Neutral	Negative
Gary	1	1	1
David	7	0	0
Jason	3	0	6
Alan	4	3	2
Marcia	23	1	2
Cathy	16	0	0
Steve	11	1	3
Chris	12	1	0
Bart	8	0	7
Ethan	2	2	8

Curricular Adaptations with Regard to CAS-S

Overall, PST typically retained lesson elements that asked students to use CAS-S. However, there were several lesson elements involving that were eliminated by PST as seen in Table 7. The majority of instances where CAS-S based lesson elements were removed was due to other non-technology issues such as time constraints or problem redundancy. There were only two cases where lesson elements were removed due to some issue with how technology was used.

Table 7: *Explanations for Elimination of CAS-S Lesson Elements*

Explanation	PST	Frequency	Lesson Component
Time Constraints	Gary	1	Homework
Problem Redundancy	Alan	1	Activity
	Ethan	3	Activity (2) Homework (1)
Question not Mathematical	Ethan	1	Activity
Cognitive Demand	Ethan	1	Homework
	Jason	1	Homework
CAS-S Without Understanding	Ethan	1	Summary
CAS-S Doesn't Promote Understanding	Bart	1	Activity

There were a number of positive adaptations to CAS-S based CPM lesson elements. Three PST, Marcia, Chris, and Steve, recognized that one of the important goals of this lesson was students' recognitions of the strengths and weaknesses of tables, graphs, and CAS-S. These PST supplemented the CPM lesson with additional questions during the activity, summary, and homework sections to bring this goal to students' attention and make sure that their focus was fixed on this important idea in places other than the summary. Cathy asked students to reflect on the solutions that students received with the CAS-S. Several PST also asked students to use other tools such as PP, graph, or table to check the solutions they were receiving on problems where it was not specifically asked for in the textbook lesson. Chris also asked students to make predictions about the solutions they were likely to get before using the CAS-S and Bart asked students to explain the procedures used by the CAS-S to find solutions. On the negative side, Jason wanted students to solve quadratic equations by hand using PP and incorporated this into his lesson, but failed to seize the opportunity for students to connect this technique to the work of the CAS-S.

Discussion and Conclusions

This study examined how a group of PST enrolled in a technology class read, evaluated, and adapted a secondary mathematics textbook lesson that contained CAS-S based elements. PST read the lesson completely and typically noted that the organizing question in the student textbook was important. They also felt that the section involving CAS-S was important, especially the section involving the command for returning answers in approximate or exact mode. Two participants did not evaluate the lesson at all, one PST evaluated from the perspective of the student, while the remaining PST evaluated from the perspective as a teacher to determine if they promoted student understanding. PST were placed into four different categories on the basis of their adaptations of the lesson: following, additive, mix, reductive and adaptive. The majority of PST made positive adaptations to the CPM lesson; only two teachers had more negative than positive adaptations. CAS-S based lesson elements were removed due to reasons that were non-technology oriented, e.g., cognitive demand. PST made a number of positive adaptations to the curriculum adding prediction to students' CAS-S work and expect that students understand what the CAS-S was doing.

PST tended to work from the curriculum that is given to them. That is, PST tended not to omit items and redevelop or adapt lesson elements. They tended to supplement small additions to the overall question changing the initial question very little such as in the case when students were asked to explain their thinking. Overall, not that many items were removed and then supplemented with a new problem. For instance, if PST were dissatisfied with a lesson element because it was redundant they tended to simply remove the question instead of rewriting the question. The education classes that PST had taken up to that point appeared to influence what they intended to use in the classroom. That is, PST frequently brought in general reform-based practices when asking students to explain their work or more mathematical based reform-oriented practices when they adapted questions to increase the cognitive demand of the tasks. In sum, when changes were made to textbook lesson elements those changes were considered to be positive.

CAS-S lesson elements were not often removed from the lesson, but when they were it was typically for reasons other than technology based. For instance, PST removed CAS-S elements if they thought they were redundant. A number of positive changes were made to the textbook lesson that built on and extended the use of CAS-S that currently appeared in the textbook lesson elements. For instance, some PST asked their hypothetical students to reflect on the strengths and weaknesses of CAS-S and other representational tools throughout the lesson. Textbooks in the U.S. have been seen as a tool to help promote the reform of mathematics education (). The results of this study with PST suggest that incorporating CAS-S into textbook resources can promote the increased use of this technology in U.S. secondary mathematics classrooms.

PST tended to not see the forest or the overarching goals to the lesson and how the individual lesson components fit together to further those goals. PST explanations for keeping lesson elements were typically directed at the specific lesson element and less about how that lesson element furthered the overall lesson goals. Notable exceptions to this were Marcia and Cathy who were able to identify one of the goals as student understanding of the strengths and weaknesses of the different representations and adapted lesson elements so that this goal became more apparent to students.

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