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The previous academic year was the third year of implementation of the new curriculum in Estonia. An important element of the curriculum is the use of mathematics software – it is now mandatory and the curriculum also specifies some learning outcomes to be achieved using mathematics software (Põhikooli..., 2011). The requirements of the curriculum can be met, to a large extent, through three software systems: T-algebra for step-by-step solving of algebra problems, GeoGebra dynamic geometry software, and Wiris computer algebra system. A number of training courses on these three software systems have been organised for teachers. GeoGebra has been the most popular application with 63% of mathematics teachers having used it for teaching, while the Wiris system has only been used by 35% of mathematics teachers (Prei, 2013).

The Wiris web-based computer algebra software Wiris can be used in Estonian schools as freeware, with the Information Technology Foundation for Education paying for the software licence. Wiris has been translated into Estonian but has not become particularly popular, despite having been in use for ten years and having been presented in a number of software training courses. A possible explanation could be the difficulty for teachers to see how the program could be meaningfully used for mathematics education. In order to fill this gap, an online course has been developed, presenting the usage possibilities of Wiris in solving problem exercises in basic school mathematics classes. The authors of the course are Sirje Pihlap and Eno Tõnisson. One aspect of the course was presentation of certain specific features of the program, in particular the instances where Wiris generates unexpected answers. Teachers were asked to give their opinions on the distinctive traits of Wiris – to what extent they interfere with the use of Wiris and how could they be utilised for educational purposes.

This presentation provides an overview of the content analysis of the opinions of 45 participating teachers regarding two specific traits of Wiris – the unconventional form of the decimal fraction and the lack of mixed fractions. In the following text, the quotes from the respondents are presented in italics.

### Using decimal point to write decimal fractions

In Estonia, decimal comma is used as the separator between the whole and fractional parts, but Wiris uses decimal point. Consequently, the conventional Estonian entry of addends would generate an incorrect result:


$$1,5+2 \rightarrow 1,7$$

Comma has a different meaning in Wiris.

The teachers were asked, **how distracting is this discrepancy?** 11 teachers of 45 believed that this discrepancy could be distracting for students, *because if they do something on their own they can get it wrong on multiple occasions* (T33) and *the students may not even notice the error* (T2). The respondents feared that *they will have to constantly remind this to the students* (25). The remaining 34 teachers stated that using point instead of comma was not

particularly distractive and *such a discrepancy simply emphasises the need to pay attention* (T35). *The students are already used to it as other programs also use point instead of comma* (T8), *the same applies to calculators and consequently it is not unexpected in this program either* (T10). One teacher (T18) suggested *using the integrated (narrow) keypad of Wiris itself, then there would be no problem.*

The teachers were asked, **would it be possible to use this discrepancy for educational purposes and how?** The responses can be divided into four categories: (1) the use of point in other countries, environments, programs; (2) discovery learning; (3) different conventions of number-writing; (4) critical assessment of answers.

- (1) The respondents believed that it would be useful for the students to observe that decimal point is also used in other places in addition to Wiris.

*There are other programs in which decimal fractions have to be entered in the same way. This is a chance to explain that one should learn about such basic things before starting to use a new program* (T24). *The students' attention can be drawn to the fact that different countries have different mathematical notations* (T26) *and that some countries use decimal point as the separator between the whole and fractional parts* (T1). *Decimal point is used in a similar way in calculators, in online banking transactions* (T30), *when noting meters and centimetres* (T35). For example, *Gerd Kanter's bronze-winning discus throw in the London Olympics was 68.03* (T40).

- (2) Discovery learning. *Initially, we could tell the students nothing about the meaning of comma and point in this program* (T4), *instead we could let them try to solve a couple of problems and discover the reason why the program does not produce the correct result* (T10). *This helps them to remember that they should use decimal point instead of comma in decimal fractions* (T4).

- (3) The example of notation of decimal fractions was seen as a good opportunity to explain to the students that comma, and other symbols, can have different meanings in different places, and that numbers can be written in different formats.

*We could discuss different forms of number notation* (T2).

*It is a possibility to demonstrate and explain to the students that symbols can have different meanings* (T11).

- (4) The respondents believed that such a discrepancy is supportive of attentiveness and understanding of the need of critical assessment of one's answers. The accuracy of a result should be assessed even if calculations are made by computer software.

*The educational use would be the situation where a student does indeed submit the answer  $1,5+2=1,7$  that would indicate that he obviously did not even look at the answer shown by the computer.. The teacher should tell that they have to check their answers.* (T33)

*This discrepancy helps to increase the level of attention.* (T25)

*This is a convenient opportunity to explain the need to check your answers – even machines should not be blindly trusted.* (T20)

The teachers were asked, **would it be reasonable as a future prospect to change the sign used as the separator between the whole and fractional parts in the Estonian cultural sphere as well?**

32 teachers thought that it would not be reasonable, 5 were undecided and 8 teachers supported the change. Those who were opposed to the change believed that traditional notation is more convenient. The change would create a lot of confusion and unreasonable expenses, for instance, to update all textbooks.

*I think that it would not be reasonable. The students are still more used to using decimal comma. Even though I have noticed that some students use point instead of comma. (T36)*

*The change of the notation convention in Estonia would create a lot of confusion. Much work would have to be done to implement the change everywhere (e.g., in banking) and to distribute information everywhere. This is why I believe that changing the separator sign is not reasonable or feasible. (T13)*

*Is the Estonian state prepared to buy new textbooks, based on the new number notation, for all the schools? As long as this has not happened, teachers will still have to explain the different notation conventions. (T45)*

Those who were undecided believed that *it does not make any difference, which sign is used as the separator between the whole and fractional parts (T2)*. The respondents stated that the discrepancy is not problematic. The supporters of the change believe that conformity with the rest of the world is important.

*In reality the notation does not make a difference but it should rather be uniform. The transition period could be very difficult but I would support the transition. (T33)*

*Communication with the rest of the world is increasing and the idea is quite promising, considering the need to prevent any mishaps caused by unawareness. (T20)*

## Mixed fractions

The addition of one to one and half in Wiris produces an incorrect result:

$$\left[ 1\frac{1}{2} + 1 \rightarrow \frac{3}{2} \right]$$

This is caused by the fact that Wiris does not support the type of mixed fraction used in Estonia. In addition to the result not being displayed as a mixed fraction, the expression that looks like a mixed fraction has a completely different meaning in Wiris (multiplication).

The teachers were asked, **how distracting is the lack of mixed fractions in Wiris?** 12 teachers thought that it was not distracting, 12 teachers said that it was not particularly or was only slightly distracting, and 17 teachers believed it to be very distracting.

*Of course, having them would be more fun but their absence is not distracting. I believe that, even without mixed fractions, Wiris is still better than nothing. (T20)*  
*It is not a problem for me. Rather, it is a chance for the students to practice conversion. (T18)*

*It is somewhat distracting. Especially the part of getting the result. I am used to demand that all improper fractions be presented as mixed fractions. (T14)*

*V5 It is distracting; after using Wiris, weaker students always started to convert mixed fractions into improper fractions before calculation.*

*V8 I think it is very distracting. For me it is important that the students learn addition/subtraction with mixed fractions, because numerators are smaller and there are fewer calculation errors, because calculations are easier. I now work with some new students who calculate with improper fractions and they have great problems, because the number get too large.*

### The teachers were asked, **would it be possible to use the lack of mixed fractions for educational purposes and how?**

The teachers' opinions could be divided into five categories: (1) repeating the meaning of mixed fraction; (2) conversion of mixed fractions into improper fractions and vice versa; (3) different tools work in different ways, varying conventions; (4) discovery learning; (5) critical assessment of answers.

- (1) The respondents stated that the absence of mixed fractions necessitates the use of its actual meaning, with the addition of the plus sign between the whole and fractional part.

This is a good opportunity for repeating the concept of mixed fractions.

*As mixed fractions have to be presented as a sum, using mixed fractions in Wiris should make the actual meaning of a mixed fraction clear to every student. (T29)*

*It could be pointed out to the students that a mixed fraction includes a notional plus sign between the whole and fractional part, not a multiplication sign like in the variable expression  $5x$ , for example. (T41)*

- (2) Due to the lack of mixed fractions in Wiris, mixed fractions have to be converted into improper fractions and improper fractions into mixed fractions, which is why this discrepancy provides a good opportunity to practice conversion skills.

*I use this as an opportunity to develop the students' conversion skills – for example, if we need to write a mixed fraction, the student mentally converts it into an improper fraction or vice versa. (T13)*

*The educational use would be the conversion of mixed fractions into improper fractions and vice versa, **but this would be completely pointless in the case of addition and subtraction, for example, and could lead to the use of incorrect techniques by the students. (T33)***

- (3) The respondents thought it important to explain that there are different tools that work in different ways (T9) and various conventions are possible.

*There are many examples of how different notations lead to different results; this would help the students to understand that if they encounter a problem that seems incorrect at first, this could be caused by a different notation. (T27)*

- (4) Discovery learning. The students themselves should find out, why Wiris does not produce the correct result and how Wiris interprets a fraction that looks like a mixed fraction.

*At first, the children enter the operation in the program and then observe the result they get. Next, they should solve the operation by hand on paper. And then they should deduce the correct form of input. (T34)*

(5) The issue of mixed fractions in Wiris provides an opportunity to understand the need of critical assessment of one's results.

*Maybe the children will remember that they should not blindly trust software. (T34)  
This is again the same issue of assessing the correctness of the answer (T20).*

The teachers were asked, **would it be reasonable as a future prospect to stop using the form of mixed fractions in the Estonian cultural sphere as well?**

Only one teacher of 45 consented to stopping the use of mixed fractions. The others were firmly opposed. A stated reason was that mixed fraction *provides a better overview of the degree of the fraction* (T10).

The teachers' opinions diverged on the issue of the best method for addition/subtraction.

*It is easier to use mixed fractions for addition/subtraction because the numbers do not get too large – it clearly helps to save time. However, some children have admitted that it is easier to subtract with improper fractions. Especially if the need to use addition from the whole part of the minuend in order to complete the operation. (T9)*

*At the same time, it would be reasonable to use improper fractions for teaching addition and subtraction and only convert the result into a mixed fraction; this would reduce the differences between the four operations, making it clearer for the children. (T10)*

There was also an option that *we should not overemphasise the need to present the result as a mixed fraction in calculation tasks, it could also be presented as an improper fraction.* (T31)

In conclusion, it could be said that even though the particular distinctive properties of Wiris are to a greater or lesser extent distracting for the teachers, they nevertheless found several opportunities for using the discrepancies for educational purposes.

### **Acknowledgements**

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