

# USING COMPUTABLE DOCUMENT FORMAT IN TEACHING MATHEMATICS

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ABSTRAKT. Interactive graphics is focused on natural senses and helps students to develop analytical thinking in the way from single perception and experience to general knowledge. Computable document format (CDF) is a file format from *Wolfram Research* Company readable in Free CDF Player. This software contains Mathematica's core, providing algorithms for visualization in 2D and 3D and functions for computation on many topics. It allows inserting interactive graphics in teaching materials by authors simply and in a comfortable way. Such textbooks in CDF can contain interactive tools for inquiry based learning with no demand on readers' programming ability.

In the introduction, I will discuss some aspects of computer-supported learning and new technologies. Thereafter I will present the CDF format and finally provide my own concept of interactive learning material for teaching statistics.

## INTRODUCTION

Experts agree that information and communication technologies should be used in education. Computer algebra systems (CAS) and Dynamic Geometry Systems (DGS) such as Matlab, Maple, Mathematica, Wolfram Alpha, TI-Nspire, Maxima or GeoGebra are increasingly strengthening their role in mathematics education.

The offer of useful applications for smart phones and tablets (such as ProCalcApp, Wolfram Alpha, Maxima on Android, GeoGebra. . . ) extends with the development of mobile devices.

Interactive graphics is focused on natural senses and helps students to develop analytical thinking in the way from single perception and experience to general knowledge.

If the teacher wants to enrich the learning materials with interactive elements, its accessibility for him and for students plays an important role in the choice of technology. He is usually forced to refer to different sources in different formats. The company Wolfram Research makes an effort to change this by releasing CDF format, which is further discussed.

### 1. INTERACTIVE FORMAT CDF

Computable Document Format is a type of electronic documents, which the company Wolfram Research released in the spring of 2011. It offers an interactive platform for visualization of content that uses the Mathematica's kernel. This allows the author to perform client-side advanced mathematical calculations in real time. It is designed for easy creation of documents with the controls of the graphic user

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interface of the operating system. User can change the parameters of defined functions in Mathematica's environment through these controls dynamically.

More detailed description of the software necessary for the reader of the document, terms of use CDF and material resources from communities around Mathematica are given in following chapters.

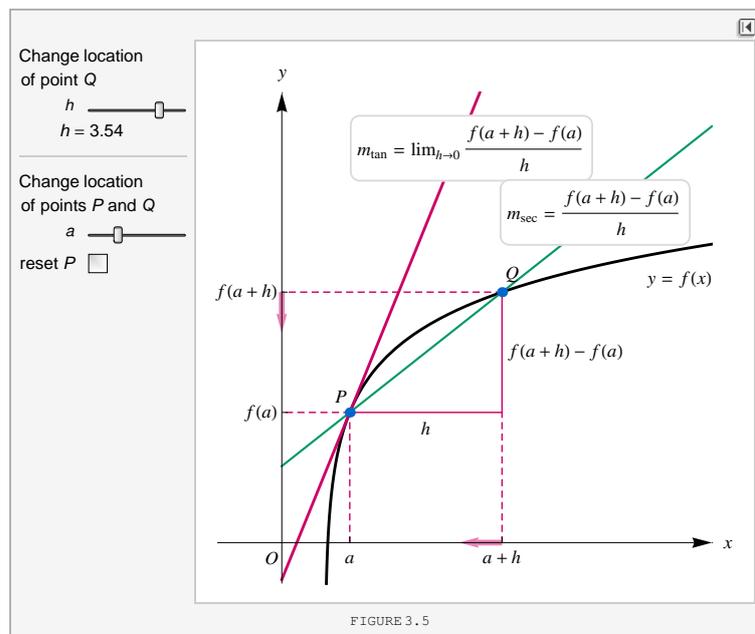


FIG. 1. Demonstration of interactive graphics from e-book [1]. Chapter *Introduction to derivative* is at the official Web site of CDF in the category of *Textbooks* for free download as a model of how e-book created in Mathematica can look like. There are texts and images distributed around as in static documents such as *pdf* along with exercises with step-by-step solutions and more.

1.1. **CDF Player** [8]. is a free downloadable software that allows reading CDF files in the standalone desktop application or through a Web browser plug-in. It is available for Windows, Mac OS and Linux. According to official statements, the priority is also support for iPads. Since the higher order is to open the CDF to everyone and everywhere in the future, cloud based services are being developed for other mobile devices also.

1.2. **Licence FreeCDF**. To create CDF documents one must have a full version of Mathematica. Distribution of authored works for non-commercial use under the name FreeCDF is licensed under Creative Commons Attribution-ShareAlike License (CC BY-SA). The license gives readers the right to free use, modification and redistribution under the same license. All FreeCDF files must contain author's name, the original title, indication of FreeCDF work and a statement of the conditions of the license. Detailed conditions of use can be found at [6] and comparison of application possibilities of FreeCDF, EnterpriseCDF (commercial deployment, e-books protected by a different license than FreeCDF ...) and Mathematica at [7].

1.3. **Wolfram Demonstrations Project** [11]. is a peer-reviewed database of thousands of interactive documents created by Wolfram Mathematica users. It is a unique collection of examples, visualizations ideas and problems sorted into many categories all the fields. It covers topics ranging from elementary school through interest in mathematics to scientific activities. All demonstrations are available for free download or run directly through CDF Player browser plug-in.

## 2. EXAMPLES

I will demonstrate using the CDF format for educational purposes by my own examples from the concept of a planned interactive textbook that could serve as an introduction to statistics.

Only a minority of students place statistics among their favorite branch of mathematics. The main reason I see is that after definition of basic terms they are overwhelmed with a number of formulas, for which they cannot imagine anything.

However, in practice, all the routine work is passed to the computer that processes all the information and calculations by calling the required functions without the user having detailed knowledge of the specific formulas and procedures.

Statistics thus appears to be ideal field of use of modern technology in teaching and new approaches to teaching. This is the motivation for the creation of interactive textbook mentioned above. In the following text, I select some key interactive elements and ideas of their insertion into the curriculum. Full text assumes a systematic breakdown, mathematical definitions and theorems, step-by-step solutions, supporting texts and of course plenty of different exercises (both classical and interactive).

2.1. **Histogram.** Using the Histogram is submitted for its intuitive implementation and widespread use in the data analysis. To construct a histogram is today in most CAS no problem. For example, the search engine Wolfram Alpha [9] will offer it automatically when you insert only the values separated by commas. All through the website, free of charge and on any device. To reduce surrounding information, it is better to insert the word „histogram“ before the data.

Assume that the basic terminology and concepts of absolute frequency, relative frequency, arithmetic mean, the harmonic mean and histogram are clarified to the students in the introductory chapters of the book.

I choose the statistical set average monthly air temperature in the Czech Republic over the last five years obtained from [3] as example. In the electronic version of CDF file a table with individual values can be hidden below the dropdown menu for a more detailed exploring. There is just a curve of the graph shown in Figure 2.

Interactive histogram of the measured values can serve students as the basic representation of the frequency distribution (see Figure ref fig: hist). The user can easily change the width  $h$  of all classes of decomposition via a single slider in the control part this demonstration.

The resulting image will dynamically change depending on the movement of the slider. The learner should observe how the choice of parameter affects the interpretation of processed data. His task is to determine what conditions must meet specific histogram to avoid excessive distortion of information. Justification of the highest frequency values in the interval below zero may be the subject of discussion at the end. It can be attributed to temperatures in winter 2013 and 2011.

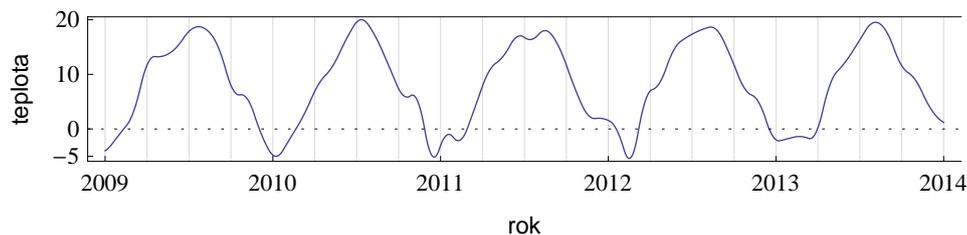


FIG. 2. The course of the average air temperature in the Czech Republic in 2009 - 2013. Simply interpolated graph as default source and histogram tool in the output is enough for this example. Table of concrete values can be clicked.

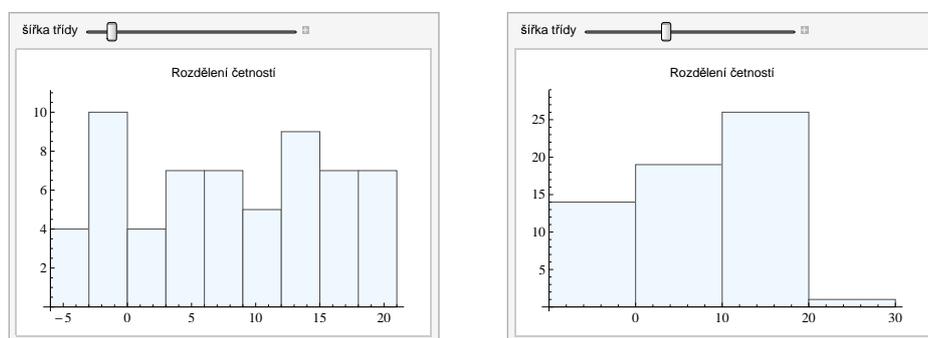


FIG. 3. Two histogram of air temperatures instances from the example given in the text when setting  $h = 3$  (left) a  $h = 10$  (right).

Based on further work with histograms concepts of variance and standard deviation can be embraced. It is possible to introduce it formally following the experience with their interpretations. An important role for effective learning plays a selection of tasks, its processing and correlation with real world.

Using random number generators we can offer easily a lot of different simulation data in the exercises to the reader. Configurable histogram of raster images can be selected as motivation and source data in some tasks.

**2.2. The probability distribution.** The next part of the textbook should naturally show the difference between discrete and continuous random variables in connection with the distribution of frequencies and then introduce the concept of probability and a random variable. The key terms in further chapters are probability function and probability density.

I think it is important to understand the different types of distribution as proven models of the probability distribution in the real world. Simply as a name for the shape of the graph (histogram), that is constantly repeated in nature and can be easily written in math language using functions. Distinguishing them as different patterns of behavior of a random variable, the rules by which each phenomenon of this variable is assigned a probability

We conclude that understanding this concept even before the formal introduction of concrete rules is crucial for further motivation to study, for abstraction introduced

concepts and formulas and especially for meaningful of the whole issue for the learner.

Therefore, the proposed e-textbook should contain enough examples of the probability distribution of random variables in dynamic modules with configurable parameters as shown in Figure 4. Generating such modules in the CDF format is not a problem even for a larger number of types of distribution.

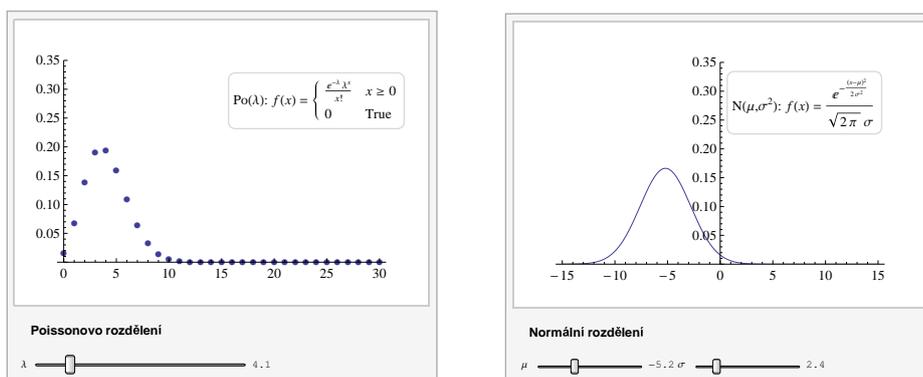


FIG. 4. An app with continuous change of parameters for Poisson and normal distributions. In the classic textbooks in there are printed three configurations in the graph mostly: full, dashed and dotted lines. In CDF we can easily move between them.

Wolfram Mathematica contains many predefined distributions and their properties, as well as programs of statistical data processing in practice (Excel, Statistica, R, Matlab ...). In effect, this means that we do not manually listing formula for density, distribution function, the mean or variance when compiling materials.

**2.3. The central limit theorem.** As a final demonstration of using CDF I present a dynamic module in Figure 5, which focuses on the operationalization of the central limit theorem. The essence of this theorem is the statement that the distribution of the sum (convolution) of mutually independent random variables  $X_i$  converges to a normal distribution.

A simple example might be the convolution of Poisson distribution. The distribution of this sum is again a Poisson distribution with parameter  $\lambda$ , which is equal to the sum of all parameters  $\lambda$ . Thus the parameter  $\lambda$  of the resulting distribution is increasing along with increasing number of input distributions. In the interactive graph (A) can be experimentally observed, that it is still easier to find the corresponding normal distribution for a larger value of this parameter.

More complicated example might be a gamma distribution, which occurs as a convolution of exponential distributions with same mean value. The resulting gamma distribution moves away from the origin again and it is close to the shape of the normal distribution with increasing number of input exponential distributions (B).

## CONCLUSION

The properties of CDF described in the article imply that this environment wants to compete static *pdf* documents in the field of presentations, articles and e-books

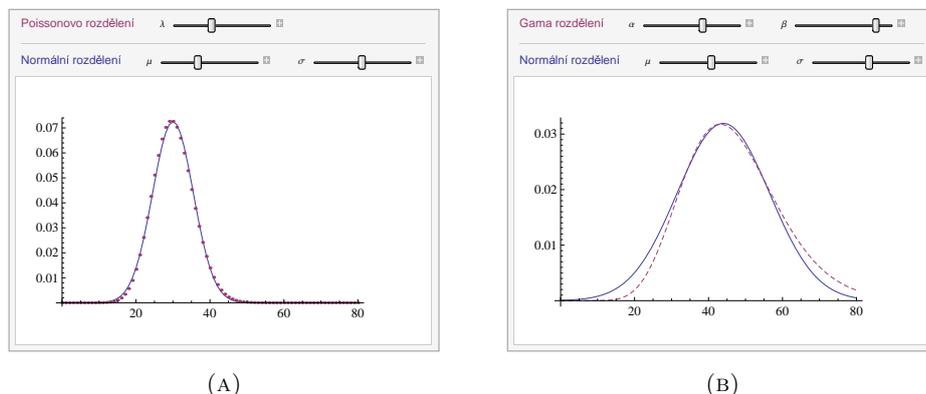


FIG. 5. (A) Poisson probability density function settings  $\lambda = 30$  and corresponding density of the normal distribution  $N(30, 5.5)$ . (B) Density of the normal distribution similar to the density of the gamma distribution (dashed) can be found not before higher values of the parameter  $\alpha$ .

with a technical focus. I would characterize it as a clean text editor with support for style sheets, interactive elements, superior calculator with advanced graphical outputs such as graphs and 3D objects in perspective, which can be rotated.

Thanks CDF Player, which is free, it has the potential significantly affect the development of teaching materials at all levels of education. It is suitable for its versatility for mathematics, physics, biology, geography and many other subjects. Probably it is most suitable where Mathematica is used more often already, i.e. at a tertiary education.

I tried to demonstrate the possibility of using the CDF on own virtual textbook of statistics examples to outline briefly the idea of implementation of such textbook. Another area of research could be observing the effects of such learning materials on learning and key competences of students from different study fields that use and teach statistics.

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## Software

- [8] <http://www.wolfram.com/cdf/>
- [9] <http://www.wolframalpha.com/>

- [10] <http://www.wolfram.com>
- [11] <http://demonstrations.wolfram.com>

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