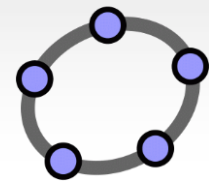


# GeoGebra3D

3.7.2014

TIME 2014

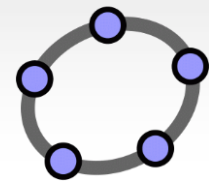
Andreas Lindner



## School of Education, Linz (Pädagogische Hochschule OÖ, Linz)



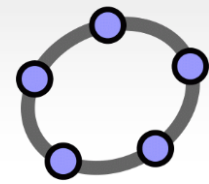
**Center for Mathematic Didactics, University Linz**



## School of Education, Linz (Pädagogische Hochschule OÖ, Linz)

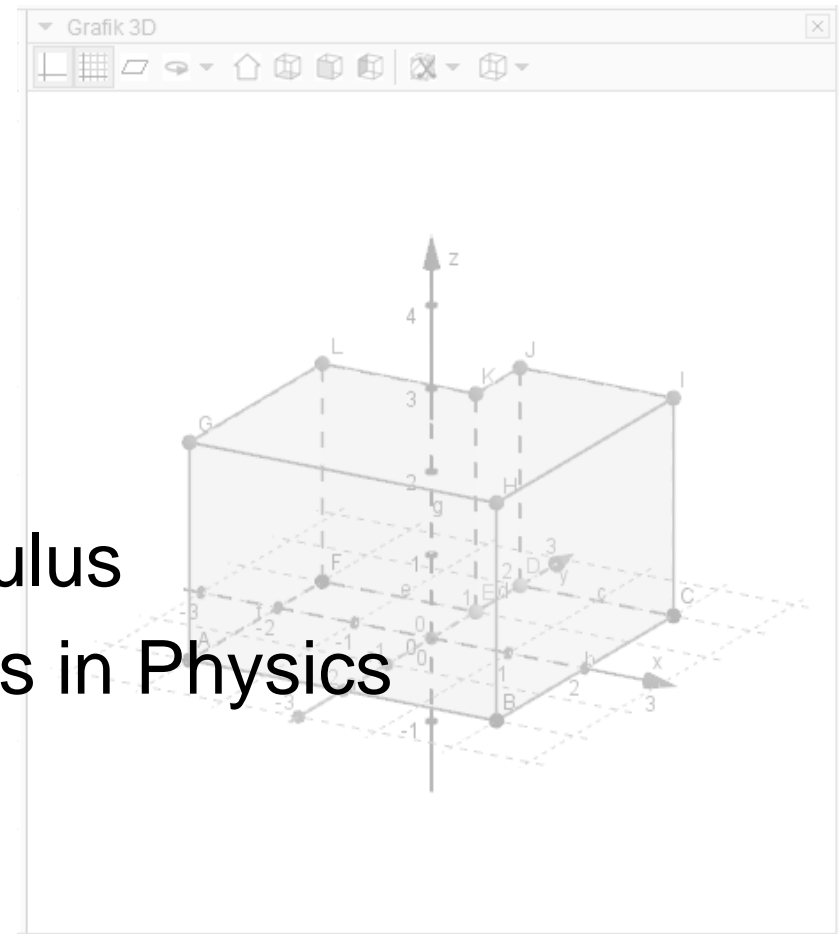


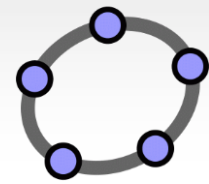
## Center for Mathematic Didactics, University Linz



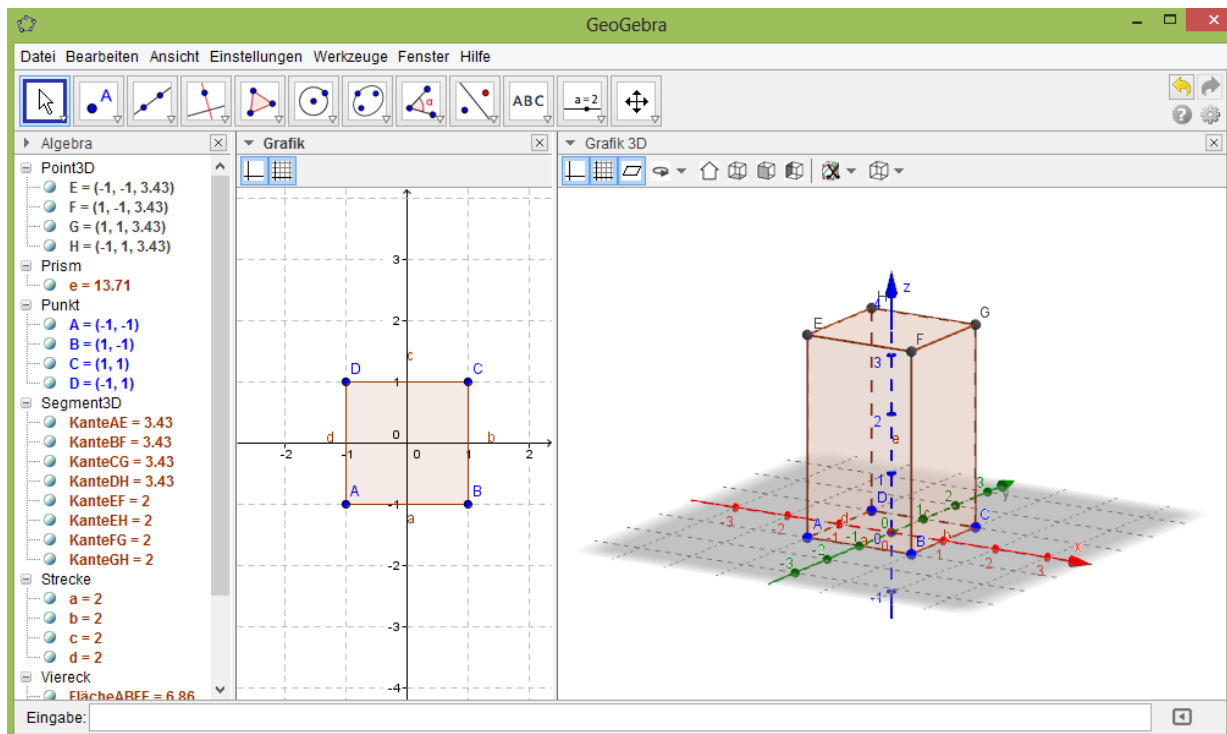
## GeoGebra3D

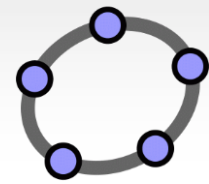
- Introduction
- Geometry
- Calculus (Analysis)
- Geometry and Calculus
- Applied Mathematics in Physics





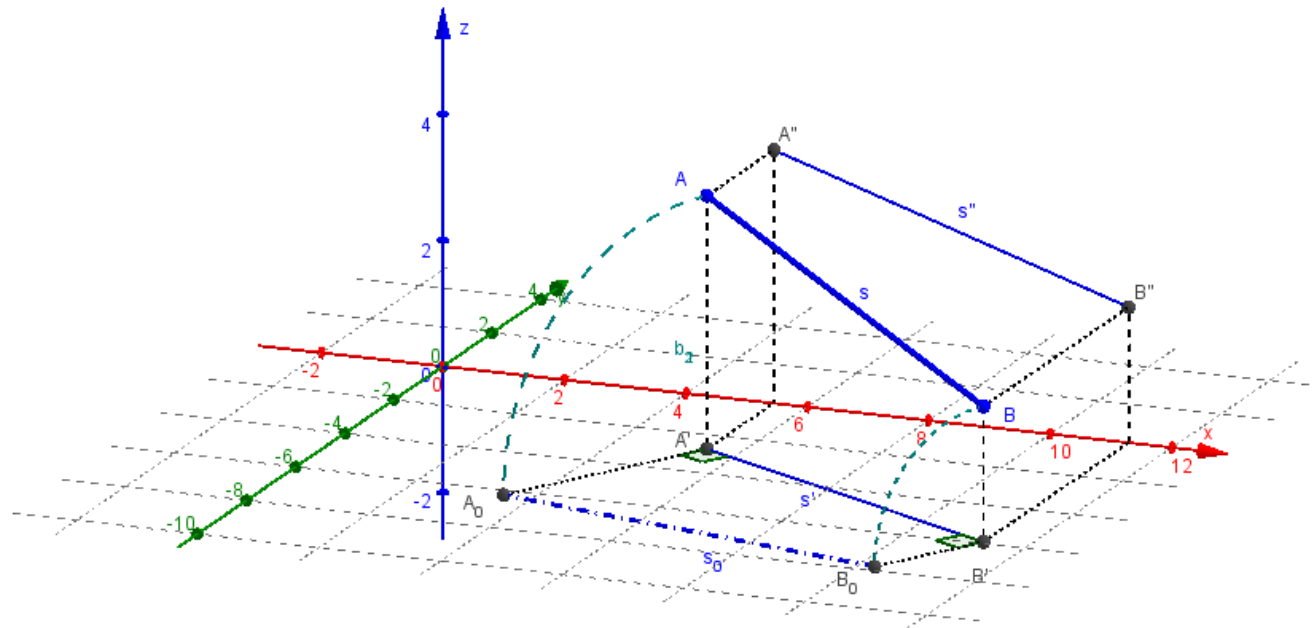
## Introduction - The Software

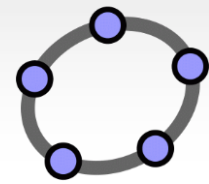




## Geometry 1

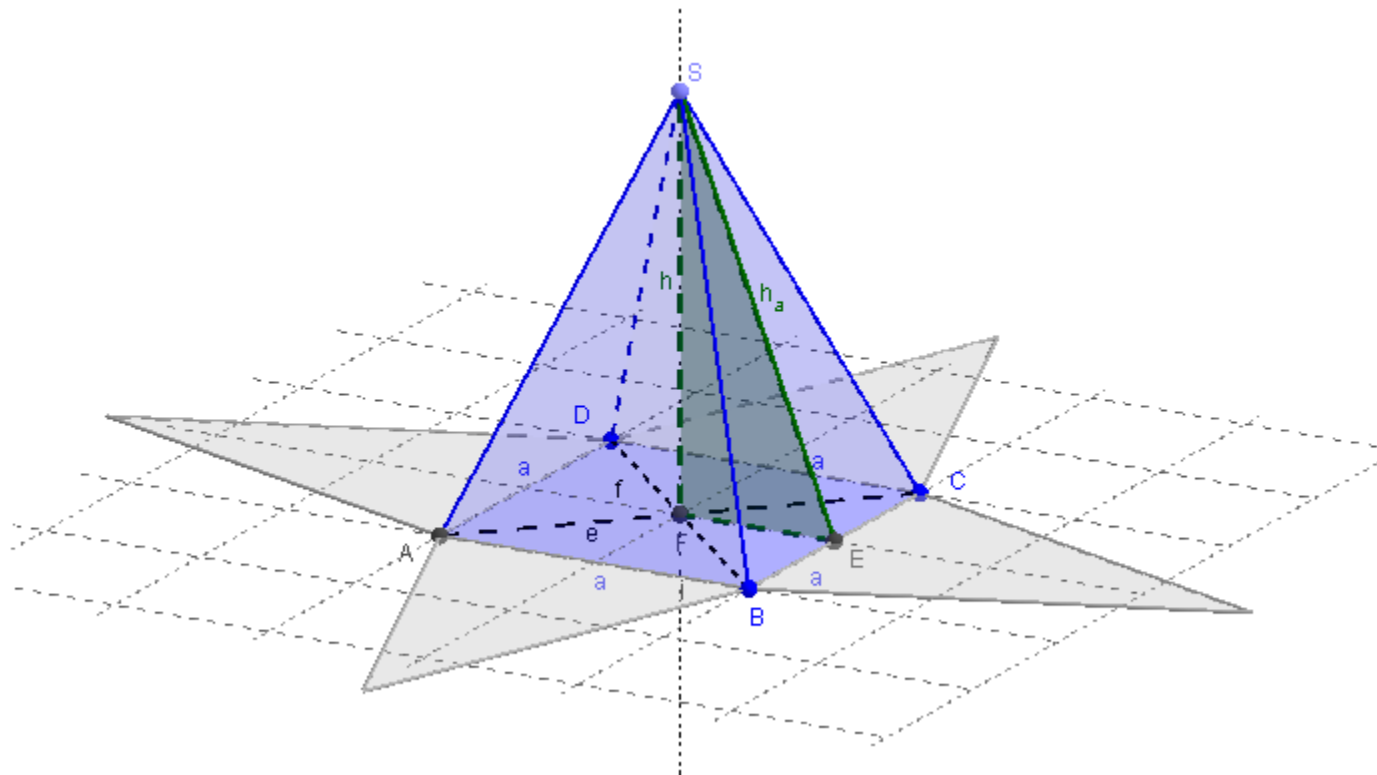
### True Length of a Line Segment

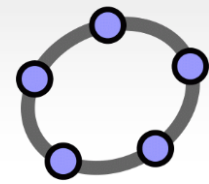




## Geometry 1

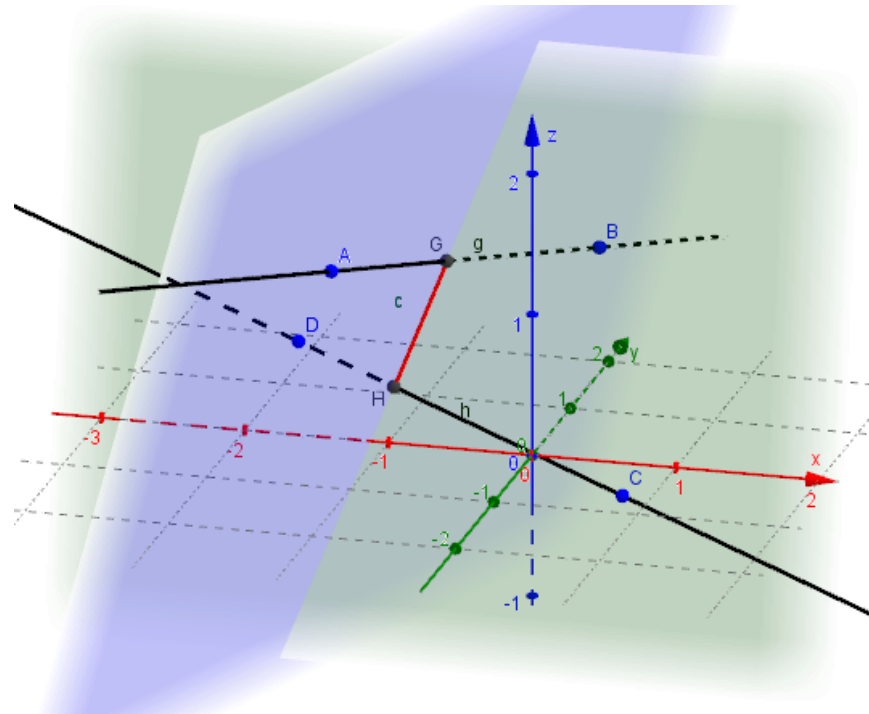
### Pyramid and Net



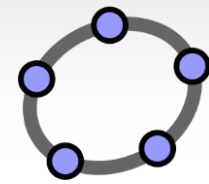


## Geometry 2

### Minimal Distance of two Skew Lines







## Geometry 3

### Analytic Geometry: Pyramid

**Dreieck**

- FlächeABS<sub>2</sub> = 70.15
- FlächeADS<sub>2</sub> = 70.15
- FlächeBCS<sub>2</sub> = 70.15
- FlächeCDS<sub>2</sub> = 70.15

**Line3D**

- f:  $X = (2, -3, 5) + \lambda(1, -8, 4)$
- g:  $X = (6, -2, 1) + \lambda(2, -1, 0)$

**Ebene3D**

- e:  $2x + y - 4z = 12$

**Point3D**

- A = (6, 1, 12)
- AB = (-12, -3, -3)
- B = (-6, 2, 9)
- BC = (4, -5, -11)
- C = (-2, -7, -2)
- D = (10, -4, 1)
- F = (2, -3, 5)
- P = (6, -2, 1)
- Q = (8, -3, 1)
- S<sub>1</sub> = (1, 5, 1)
- S<sub>2</sub> = (3, -11, 9)

**Pyramid**

- e = 486

**Segment3D**

- KanteAS<sub>2</sub> = 12.73
- KanteBS<sub>2</sub> = 12.73
- KanteCS<sub>2</sub> = 12.73
- KanteDS<sub>2</sub> = 12.73

**Vector3D**

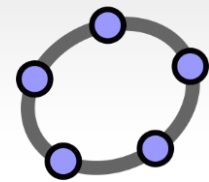
- a = 12.73
- b = 12.73
- c = 12.73
- d = 12.73
- $n = \begin{pmatrix} -18 \\ 144 \\ -72 \end{pmatrix}$

**Viereck**

- Viereck1 = 162

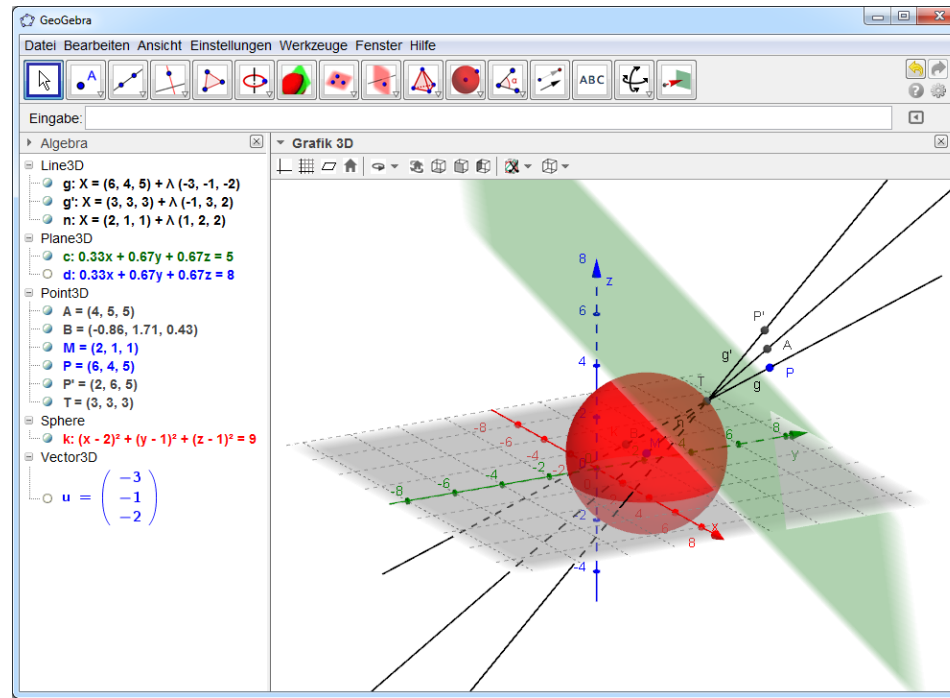
**T**

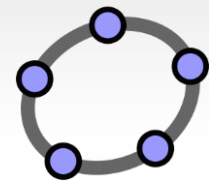
- a)
- AB=B-A  
→  $AB := (-12, -3, -3)$
- BC=C-B  
→  $BC := (4, -5, -11)$
- 
- Länge|BC|  
→ Länge:  $9\sqrt{2}$
- Skalarprodukt[AB,BC]  
→ Skalarprodukt: **0**
- Beide Seiten sind gleich lang und schließen einen
- b)
- $F = (A+C)/2$   
→  $F := (2, -3, 5)$
- $n = \text{Kreuzprodukt}[AB, BC]$   
→  $n := (-18, 144, -72)$
- $S_1 = F + 9 \cdot \text{Einheitsvektor}[n]$   
→  $S_1 := (1, 5, 1)$
- $S_2 = F - 9 \cdot \text{Einheitsvektor}[n]$   
→  $S_2 := (3, -11, 9)$



## Geometry 4

### Analytic Geometry: Reflexion of a Ray





## Geometry 5

### Analytic Geometry: Intersection of 3 Planes

1  $\epsilon_1: a_1x+b_1y+c_1z=d_1$   
 $\approx 2x+y+z=-2$

2  $\epsilon_2: a_2x+b_2y+c_2z=d_2$   
 $\approx 2x+y-z=-1.4$

3  $\epsilon_3: a_3x+b_3y+c_3z=d_3$   
 $\approx 1.5x-4y+z=1$

4  $Loes=Lösungen[(\epsilon_1,\epsilon_2,\epsilon_3),(x,y,z)]$   
 $\rightarrow \begin{pmatrix} -11 & -103 & -3 \\ 19 & 190 & 10 \end{pmatrix}$

5  $S=(Element[Loes,1],Element[Loes,2])$   
 $\approx (-0.58, -0.54, -0.3)$

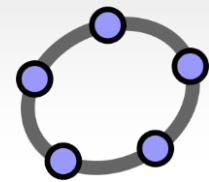
6

$a_1 = 2$	$b_1 = 1$	$c_1 = 1$	$d_1 = -2$
$a_2 = 2$	$b_2 = 1$	$c_2 = -1$	$d_2 = -1.4$
$a_3 = 1.5$	$b_3 = -4$	$c_3 = 1$	$d_3 = 1$

$E_{1,2}: X = (-0.68, -0.34, -0.3) + \lambda(-0.33, 0.67, 0)$

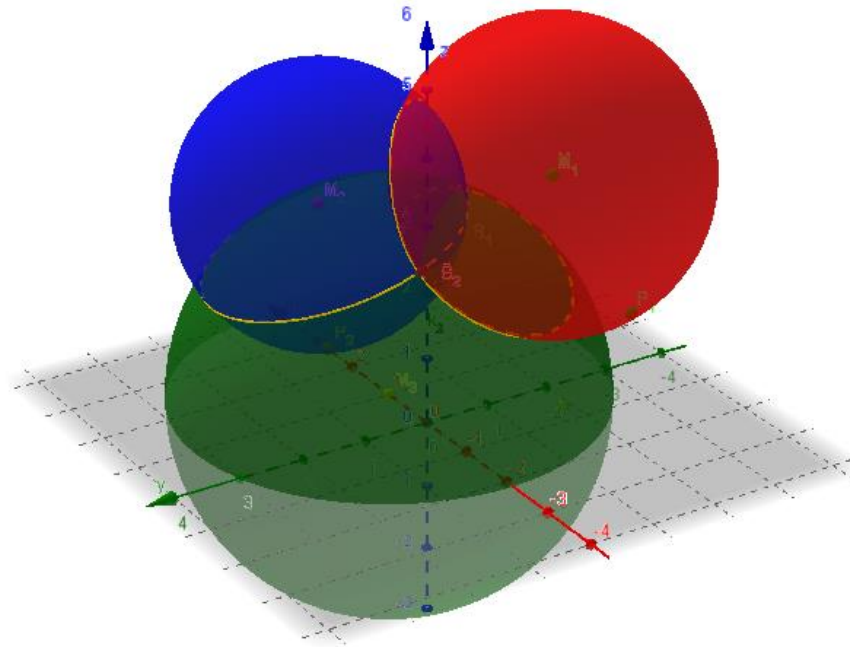
$E_{2,3}: X = (-0.4, -0.34, 0.25) + \lambda(-0.28, -0.33, -0.88)$

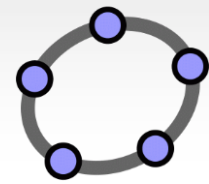
$E_{1,3}: X = (-0.59, -0.54, -0.28) + \lambda(0.47, -0.05, -0.88)$



## Geometry 6

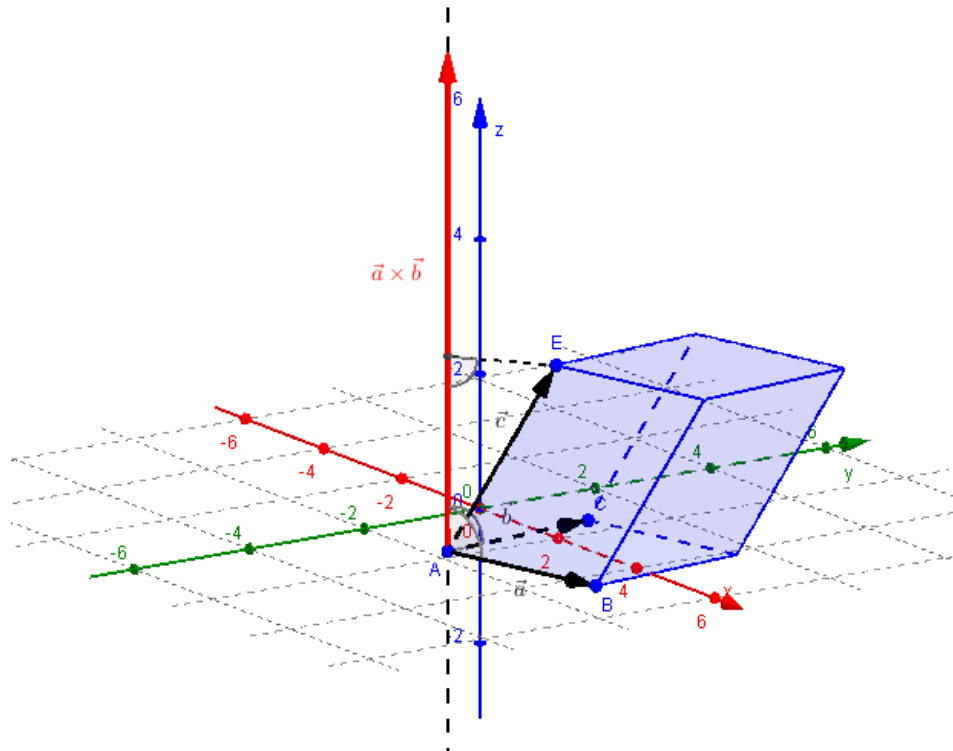
### Analytic Geometry: Intersection of 3 Spheres

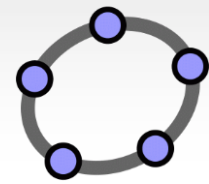




## Geometry 7

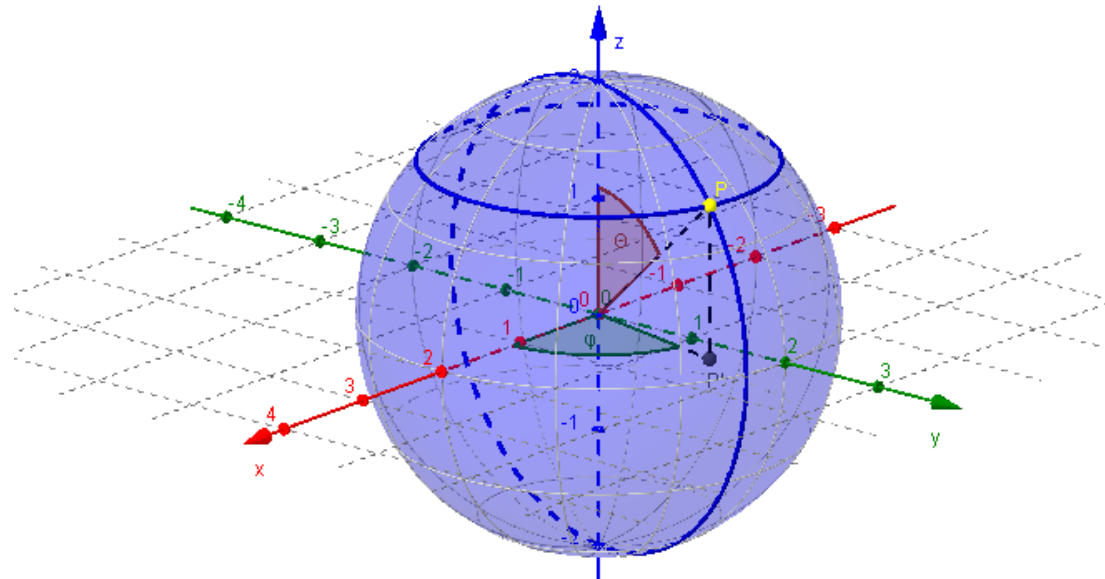
### Analytic Geometry: Parallelepiped

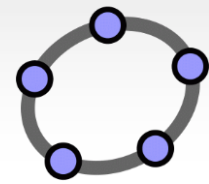




## Geometry 8

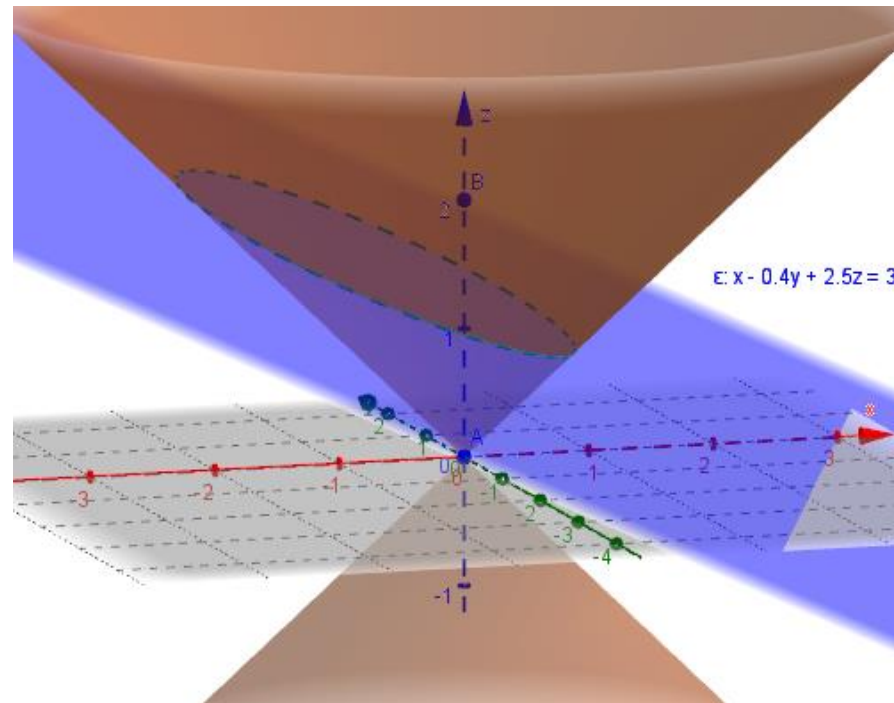
### Spherical Coordinates

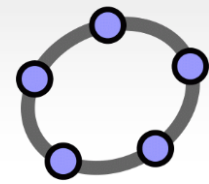




## Geometry 9

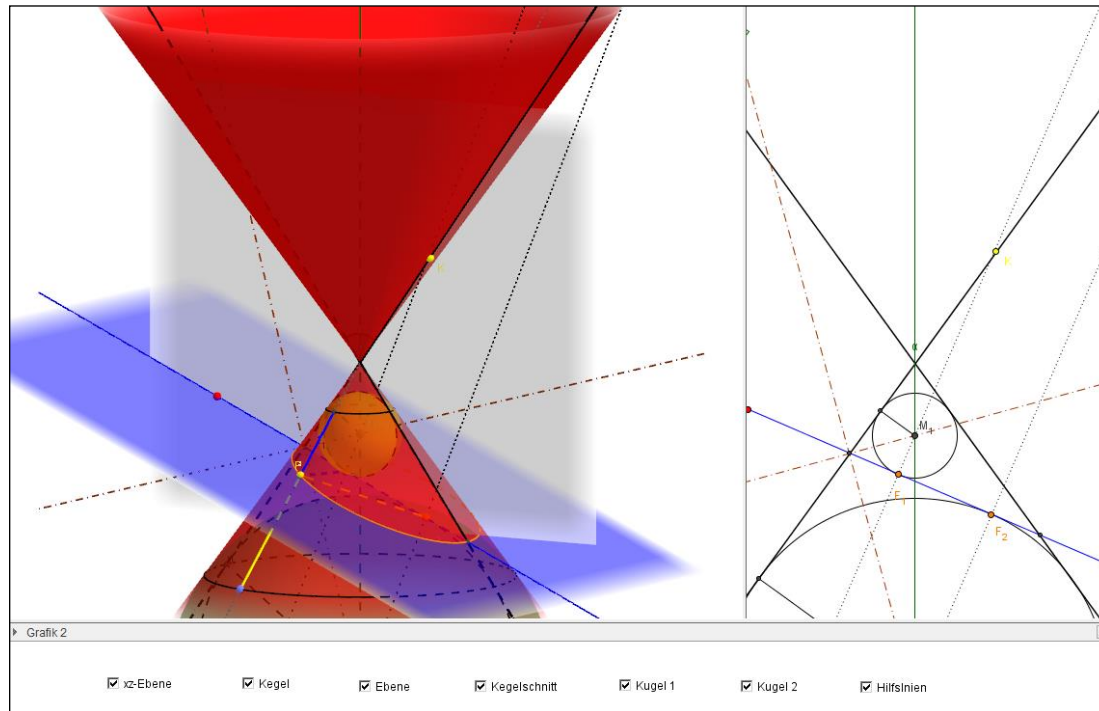
### Conics: Plane and Cone



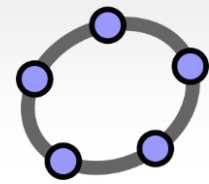


## Geometry 10

### Dandelin's Spheres

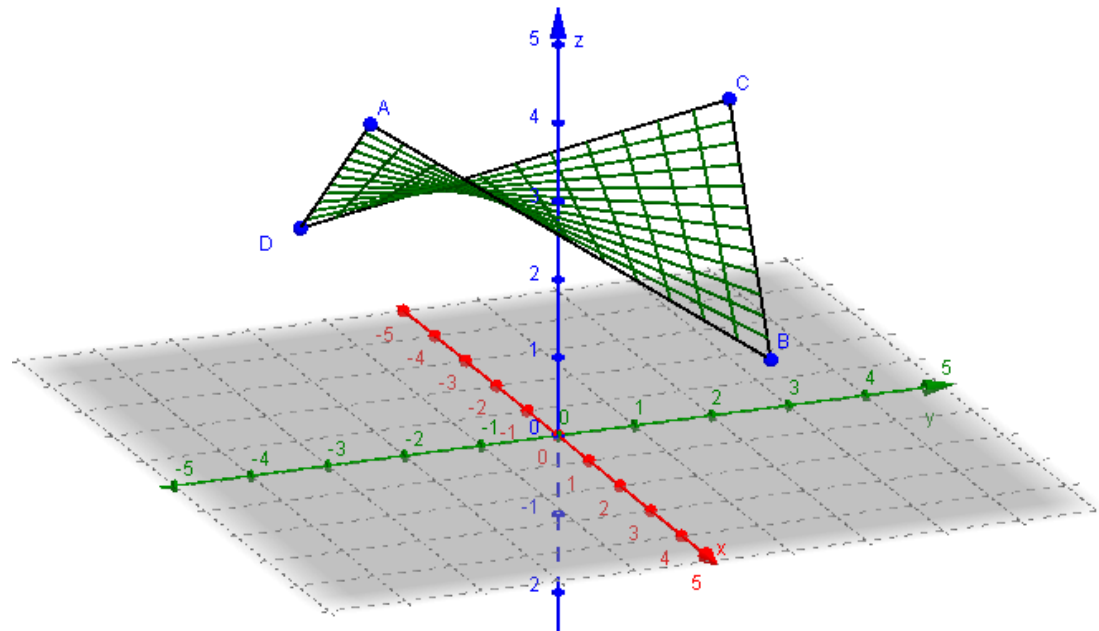


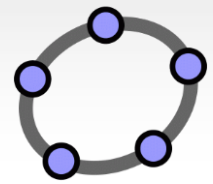




## Geometry 11

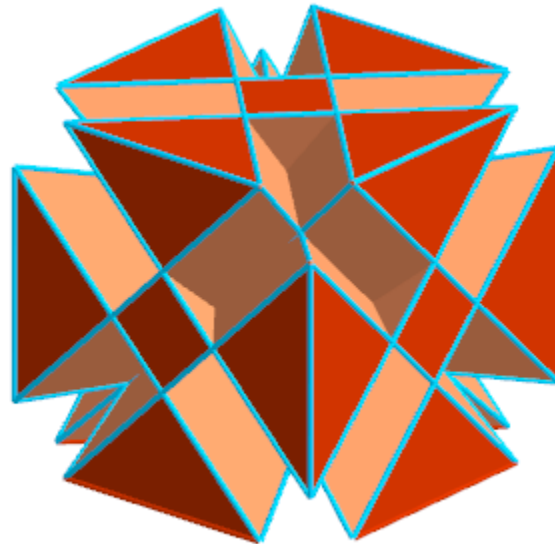
### Surface

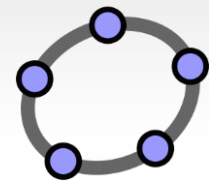




## Geometry 12

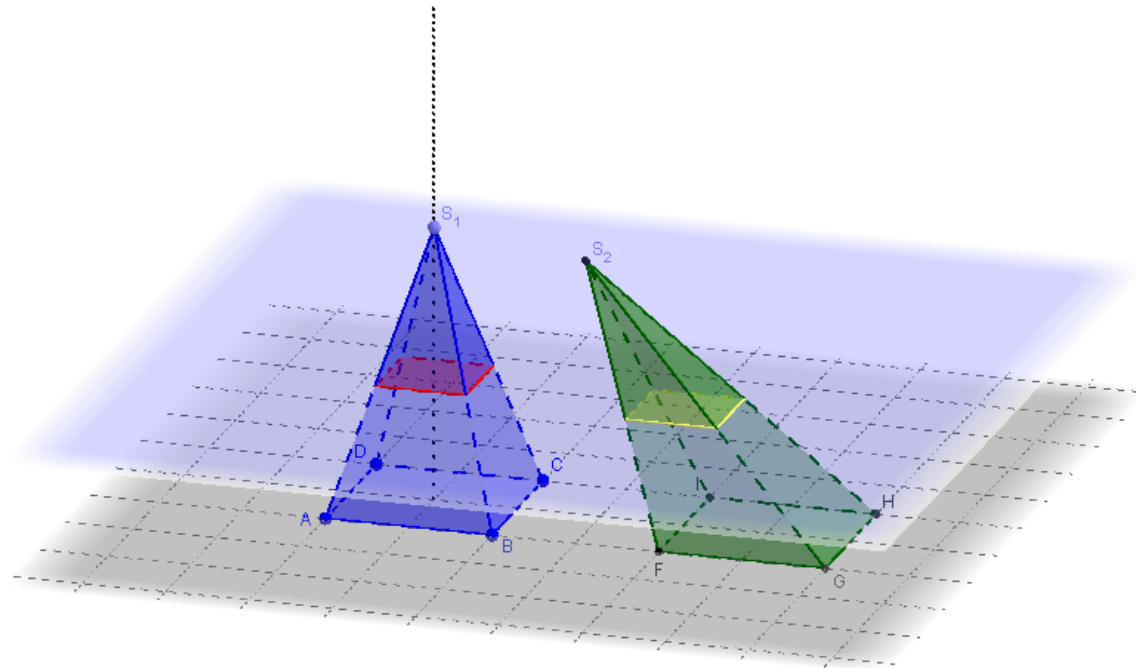
Some more Geometry: Mariagami

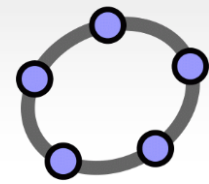




## Calculus 1

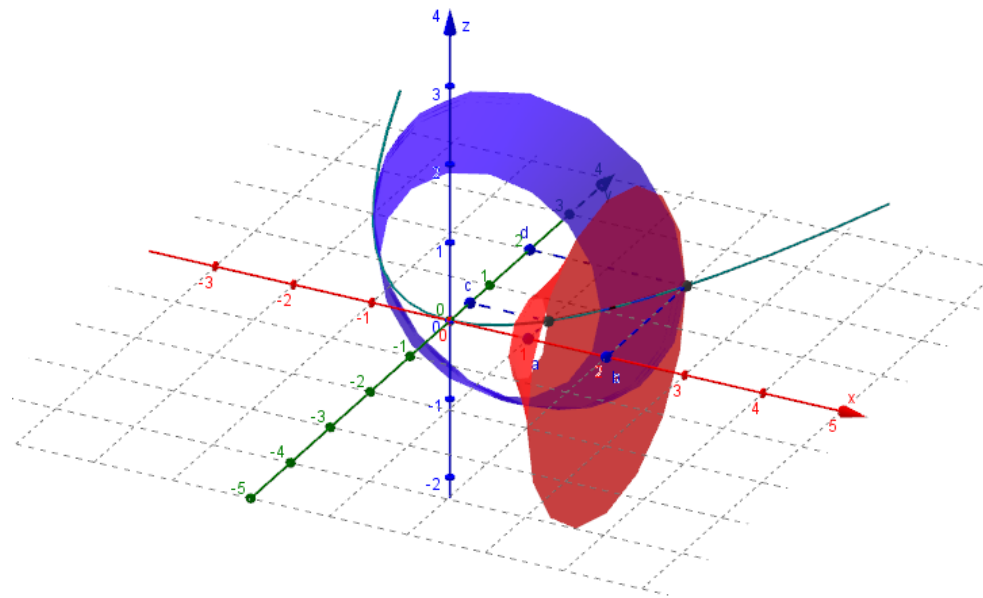
### Principle of Cavalieri

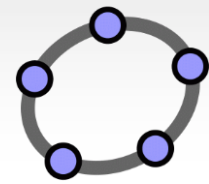




## Calculus 2

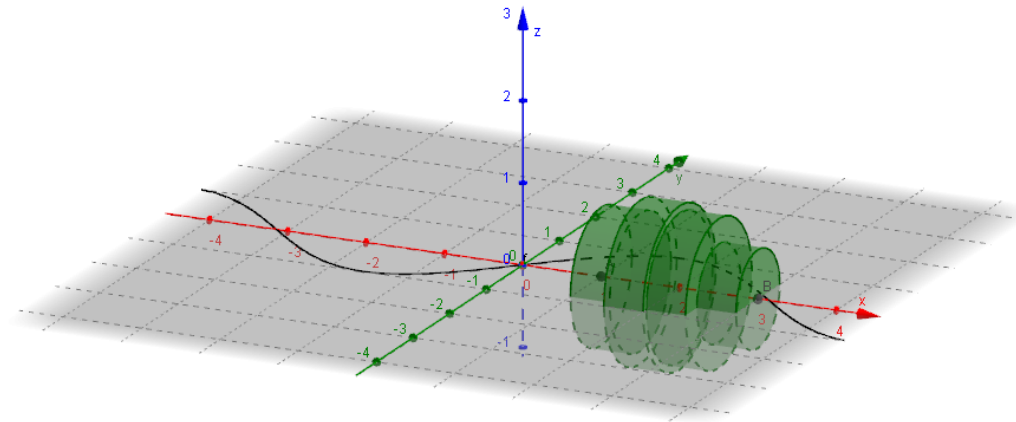
### Solids of Revolution





## Calculus 3a

### Calculation of Volumes



Grafik 2

n = 4  
Anzahl der Unterteilungen

Zylinder ein/aus

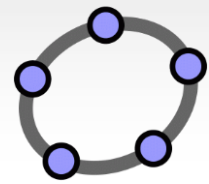
Funktion f(x) =

von a =  bis b =

Rotationskörper ein/aus

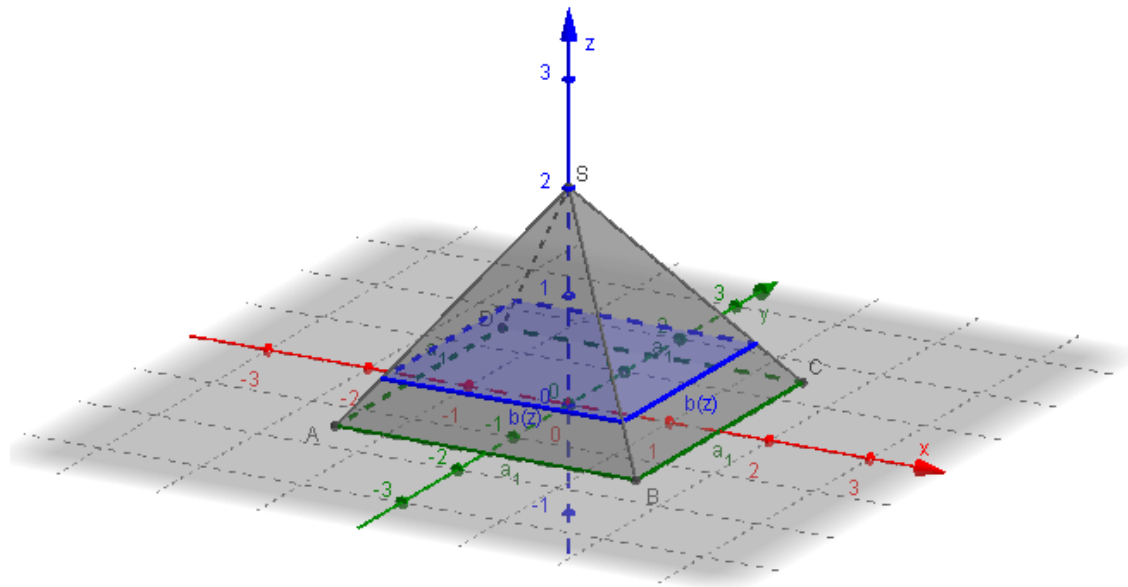
$$V_K = \sum_{i=1}^n V_i = \sum_{i=1}^n \pi \cdot (f(x_i))^2 \cdot \Delta x = 4.54$$

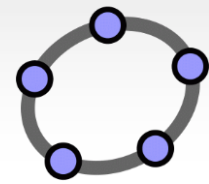
$$V = \pi \cdot \int_1^3 (f(x))^2 dx = 4.08$$



## Calculus 3b

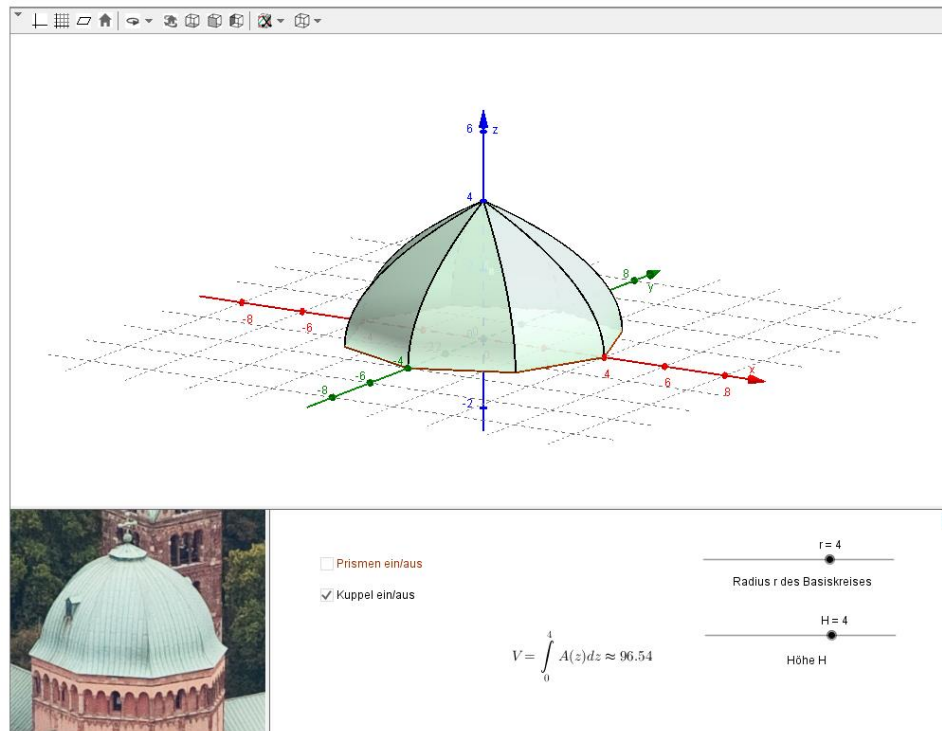
### Calculation of Volumes

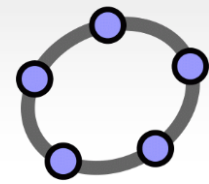




## Calculus 3c

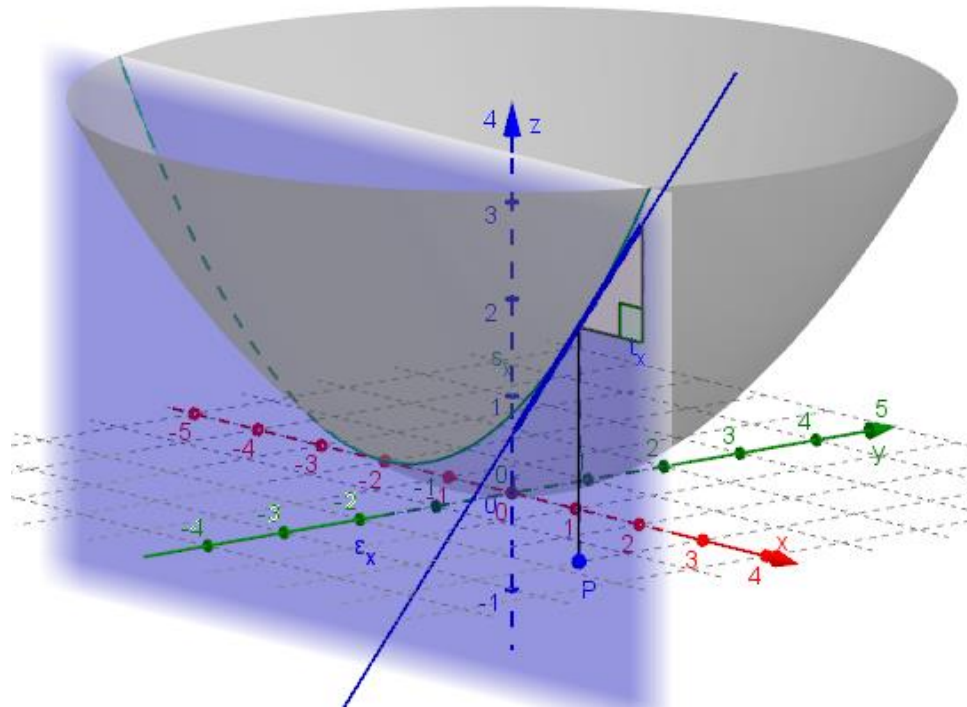
### Calculation of Volumes



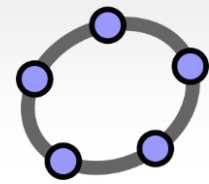


## Calculus 4

### Tangents of an Area

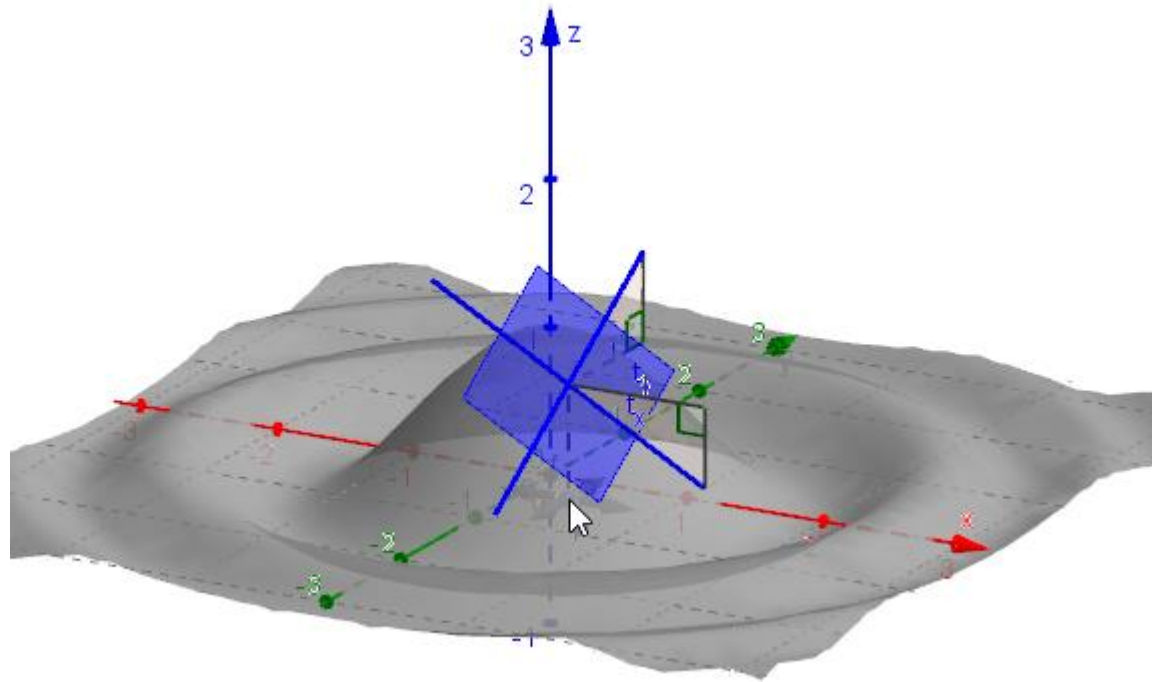


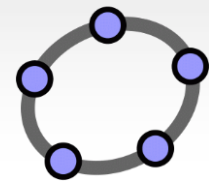




## Calculus 5

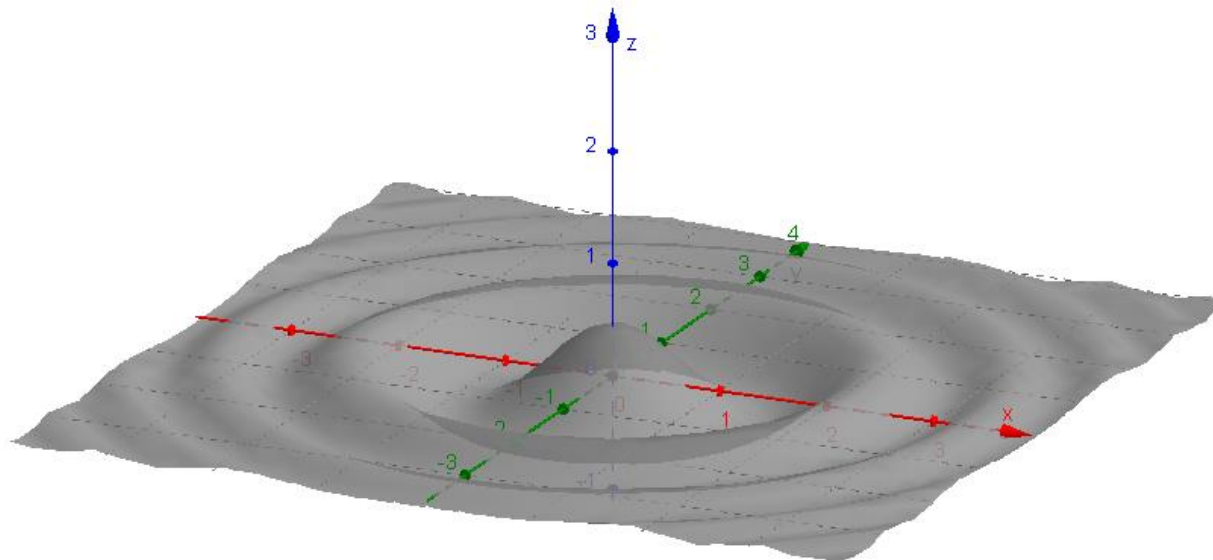
### Tangent Plane of an Area

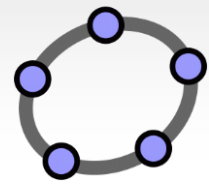




## Calculus 6

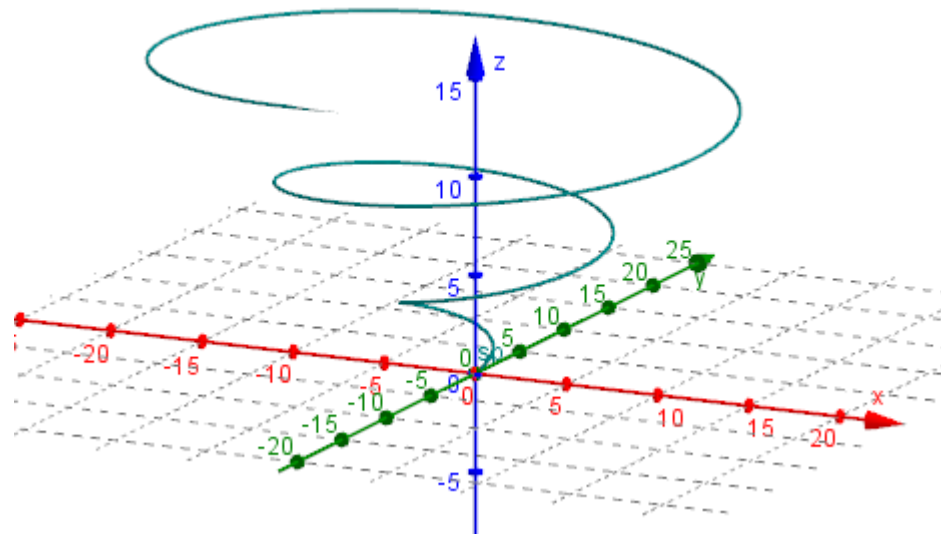
### Surface in Motion

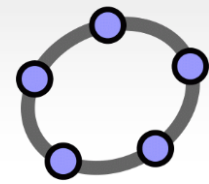




## Calculus 7

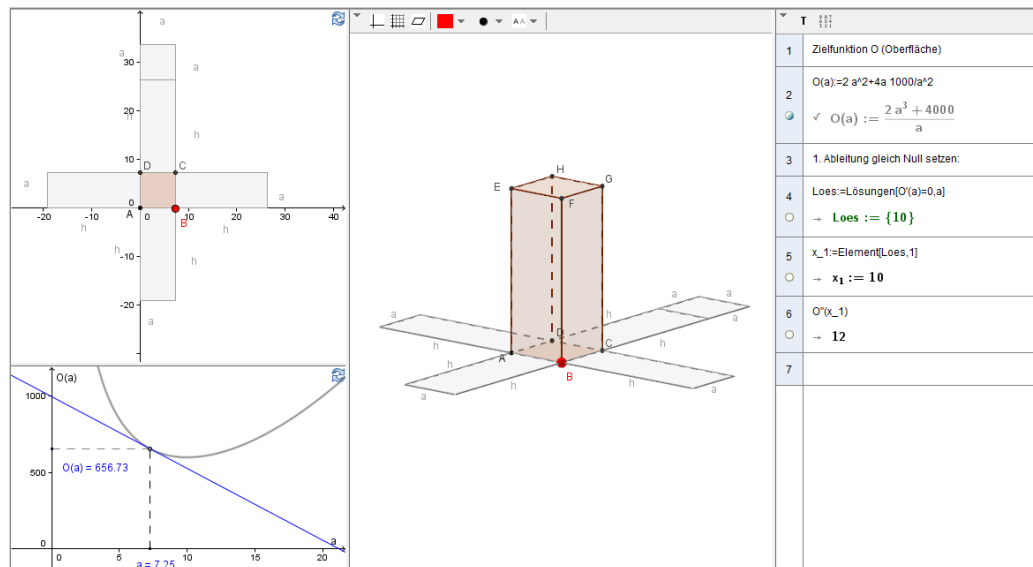
### Spiral Curve

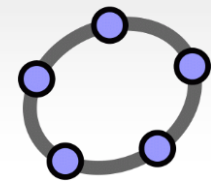




## Geometry and Calculus 1

Optimization: Minimal Surface of a Cuboid (Prism)  
(2D-View, 3D-View and CAS)

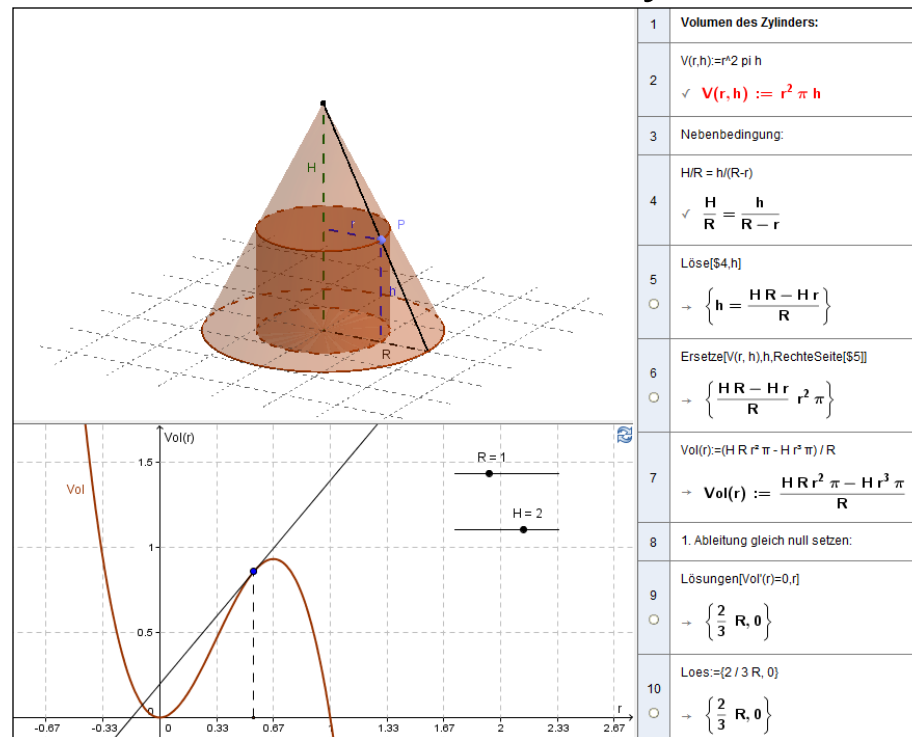


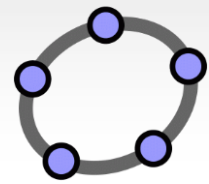


## Geometry and Calculus 2a

### Optimization: Maximal Volume of a Cylinder

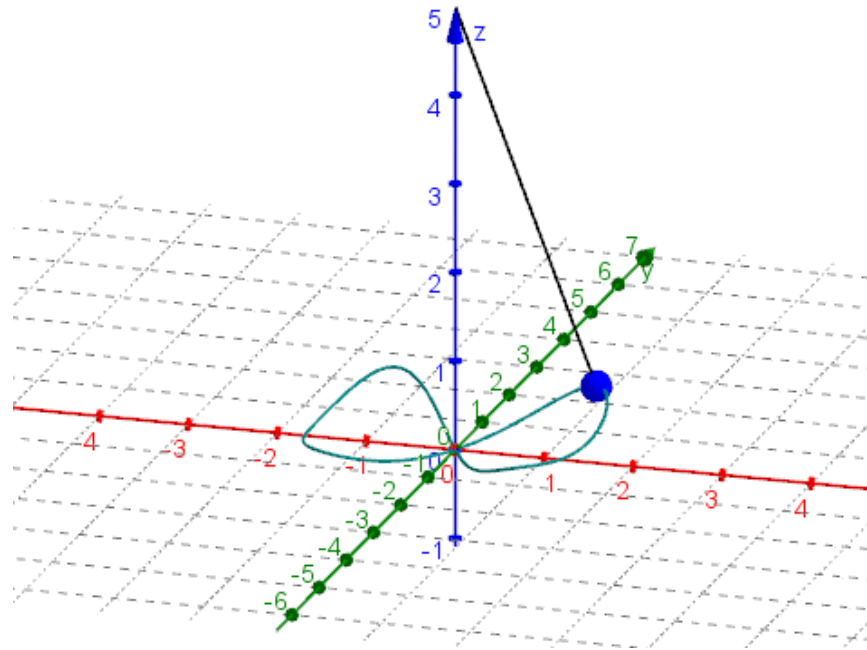
- 2D-View
- 3D-View
- CAS

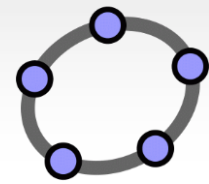




## Applied Mathematics in Physics 1

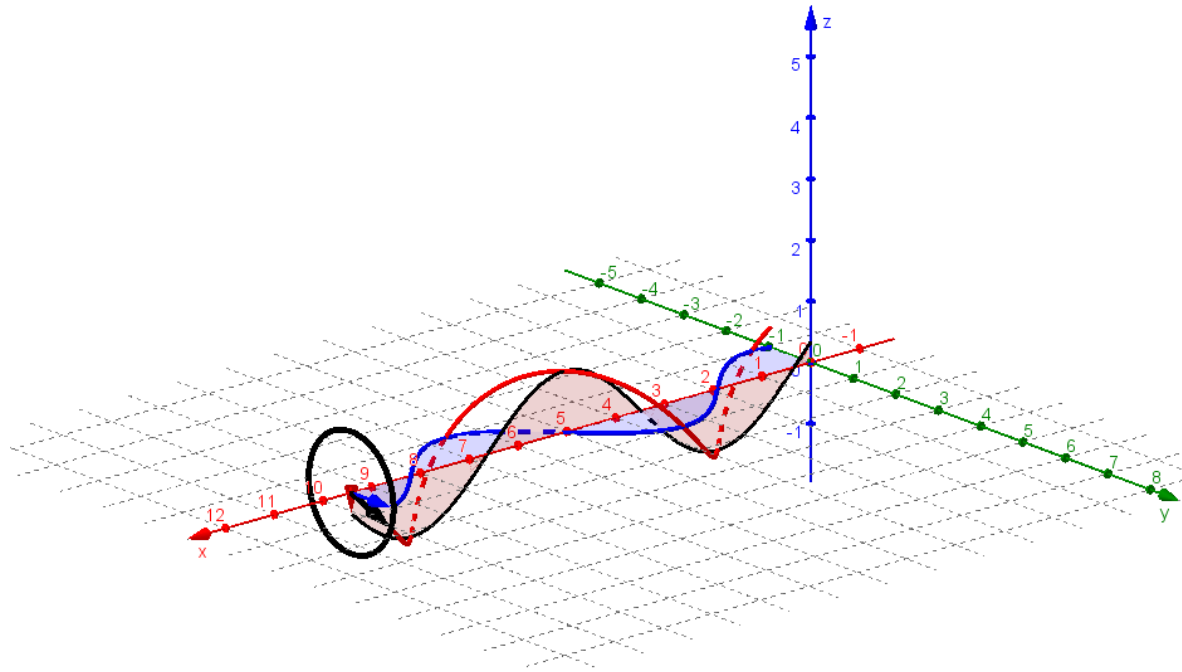
### Pendulum and Lissajous-Figures

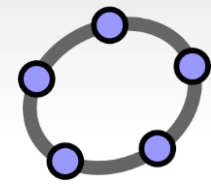




## Applied Mathematics in Physics 2

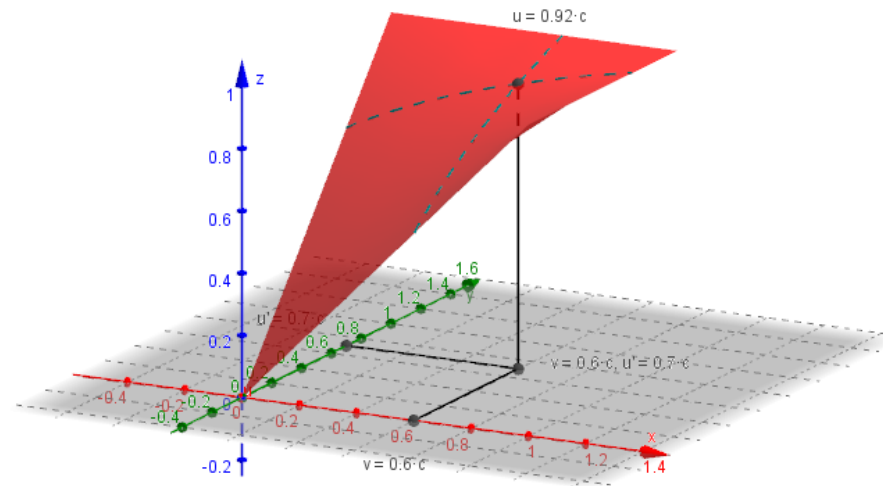
### Circular Polarized Waves





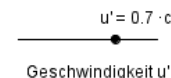
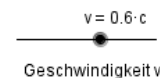
## Applied Mathematics in Physics 3

### Theory of Relativity: Addition of Velocities

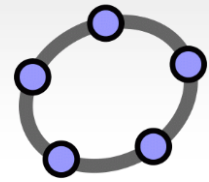


► Grafik 2

$$u = \frac{v + u'}{1 + \frac{v \cdot u'}{c^2}} = 0.92 \cdot c$$







- GeoGebra [www.geogebra.org](http://www.geogebra.org)
- GeoGebraTube [tube.geogebra.org](http://tube.geogebra.org)
- Austrian GeoGebra Institute [www.geogebra.at](http://www.geogebra.at)
- Contact: Andreas Lindner [andreas.lindner@ph-ooe.at](mailto:andreas.lindner@ph-ooe.at)